N73-19465

NASA CR-120967

MODERN METHODOLOGY OF DESIGNING TARGET RELIABILITY INTO ROTATING MECHANICAL COMPONENTS

by Dimitri B. Kececioglu and Louie B. Chester

THE UNIVERSITY OF ARIZONA
College of Engineering
Engineering Experiment Station

Prepared for
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
NASA Lewis Research Center
Grant NGR 03-002-044

1. Report No. NASA CR-120967	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle MODERN METHODOLOGY OF DESIGNING TARGET RELIABILITY INTO ROTATING MECHANICAL COMPONENTS		5. Report Date January 31, 1973
		6. Performing Organization Code
7. Author(s) Dimitri B. Kececioglu and Louie B. Chester		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address The University of Arizona Tucson, Arizona 85721		11. Contract or Grant No. NGR 03-022-044
		13. Type of Report and Period Covered Contractor Report
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code
15. Supplementary Notes Project Manager, Vincent R. Lalli, Spacecraft Technology Division, NASA Lewis Research Center, Cleveland, Ohio		

16. Abstract

Theoretical research done for the development of a methodology for designing specified reliabilities into rotating mechanical components is referenced to prior reports submitted to NASA on this research. Experimentally determined distributional cycles-to-failure versus maximum alternating nominal strength (S-N) diagrams, and distributional mean nominal strength versus maximum alternating nominal strength (Goodman) diagrams are presented. These distributional S-N and Goodman diagrams are for AISI 4340 steel, R_c 35/40 hardness, round, cylindrical specimens 0.735 in. in diameter and 6 in. long with a circumferential groove 0.145 in. radius for a theoretical stress concentration K_t = 1.42 for Phase I research and 0.034 in. radius for a K_t = 2.34 for Phase II research. The specimens are subjected to reversed bending and steady torque in specially built, three complex-fatigue research machines. Based on these results, the effects on the distributional S-N and Goodman diagrams and on service life of superimposing steady torque on reversed bending are established, as well as the effect of various stress concentrations. In addition a computer program for determining the three-parameter Weibull distribution representing the cycles-to-failure data, and two methods for calculating the reliability of components subjected to cumulative fatigue loads are given.

17. Key Words (Suggested by Author(s))		18. Distribution Statement
Mechanical Reliability	Distributional S-N Diagrams	Unclassified - unlimited
Design by Reliability		
Probabilistic Design	Distributional Goodman Diagram	
Cycles-to-Failure Distributions		

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	xiii + 183	

* For sale by the National Technical Information Service, Springfield, Virginia 22151

NASA CR - 120967

FINAL REPORT

MODERN METHODOLOGY OF DESIGNING TARGET RELIABILITY INTO ROTATING MECHANICAL COMPONENTS

by Dimitri B. Kececioglu and Louie B. Chester

THE UNIVERSITY OF ARIZONA

College of Engineering

Engineering Experiment Station

prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

NASA Lewis Research Center

Grant NGR 03-002-044

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

LIST OF FIGURES

LIST OF FIGURES

LIST OF FIGURES

LIST OF FIGURES

LIST OF TABLES

LIST OF TABLES

LIST OF TABLES

SUMMARY

This research under NASA Grant NGR 03-002-044 was initiated in 1965, and included the theoretical research for the development of an effective methodology for designing specified reliabilities into mechanical components, and experimental research to develop three fatigue reliability research machines which can apply a reversed bending moment combined with a steady torque to round, rotating, ungrooved and grooved specimens.

Phase I of the experimental research program, initiated in 1967, included the generation of distributional cycles-to-failure versus alternating bending stress (S-N) diagrams and of distributional Goodman strength diagrams for specimens made of AISI 4340 steel, R_c 35/40 hardness, and having a circumferential groove which provides a theoretical stress concentration factor, K_t, of 1.42.

Phase II of the experimental research program was identical to that of Phase I except that the specimen groove provided a theoretical stress concentration factor of 2.34, and was initiated in September 1970.

Phase II results are compared with Phase I results in this report and the effects of superimposing a steady torque on reversed bending on the distributional S-N and Goodman diagrams are presented, as well as the effect of different K_t's. Such distributional data has to be generated to enable the designing of specified, target reliabilities into components.

A methodology and a computer program were developed for the generation of finite-life, distributional Goodman diagrams. This methodology provides the capability for optimizing a design to achieve a target reliability for a specified component life.

A FORTRAN computer program was developed to estimate the parameters of the three-parameter Weibull distribution representing the cycles-to-failure data, and to perform Chi-Squared and Kolmogorov-Smirnov goodness-of-fit tests. The program also calculates the predicted component life for a specified reliability.

Also a study was made of the cumulative fatigue theory found in the current literature, and a number of methods for making cumulative fatigue reliability predictions were explored. The most promising method of conditional probabilities is proposed for further study and verification through a cumulative fatigue test program.

1. INTRODUCTION

1.1 Background

The research under NASA Grant NGR 03-002-044, initiated at The University of Arizona in September 1965, included theoretical research for the development of an effective methodology for designing specified, target reliabilities into mechanical components and experimental research to generate distributional, statistical S-N and Goodman diagrams to provide the design data needed in support of this theoretical research.

During the first reporting period a basic methodology for design by reliability in combined-stress fatigue with time dependent strength distributions was developed. Mathematical methods in dealing with the functions of random variables involved were discussed. Concurrently, a supporting experimental fatigue research program was planned. It was found that there were no research machines available which could apply to round, rotating test specimens a combination of a reversed bending moment and a constant torque. As a result three machines, similar in principle to the Mabie and Gjesdahl [1] test machines were designed and built at The University of Arizona. A complete discussion of the design and development of the test machines was given in the report to NASA CR - 72836 [2].

During the second reporting period, the operational capability of the first machine was proven and two additional machines were fabricated. The design and development of, and the results obtained from, these three machines were presented in the report to NASA CR - 72838 [3]. During the

same period calibration constants were determined for each machine so that the nominal bending stress and the shear stress in the specimen groove can be calculated. The calibration procedure, data, analysis, and constants were presented in the report to NASA CR - 72839 by Kececioglu and McConnell [4]. Calibration constants are needed because the bending and shear stresses cannot be monitored directly. Strain gages mounted in a specimen groove would be destroyed when the specimen failed. Hence it was necessary to mount strain gages on the specimen holders instead of on the specimens to monitor the bending and shear stresses. This necessitated the determination of calibration constants and equations to relate the strain at the strain gages to the nominal stresses in the specimen groove.

Another report by Kececioglu and Smith, NASA CR - 72835 [5], presented all of the experimental data generated up to June 30, 1970. Included were the reduction of the data, the application of the design by reliability methodology, the conversion of the cycles-to-failure data to stress-to-failure distributions, and the development of three-dimensional Goodman fatigue strength surfaces, which is the ultimate form of the reduced data for direct use by designers. During this period computer programs for use in the reduction of the data were developed and refined. The most important were program STRESS and program CYTOFR. Program STRESS calculates the bending and shear stresses applied to each specimen, the ratio of alternating to mean stress, and the mean and standard deviation of each stress for each test series. It also calculates the cycles-to-failure from the times-to-failure data recorded during the fatigue tests. Program CYTOFR calculates the mean and standard deviation of the cycles-to-failure for the normal and the lognormal distributions which approximate the true distribution of the data. It then calculates the coefficients of skewness

and kurtosis and applies the Chi-Squared and the Kolmogorov-Smirnov goodness-of-fit tests to determine which distribution provides a better fit to the data.

1.2 Experimental Research

Schematic diagrams of the NASA Complex-Fatigue Research are shown in Figs. 1 and 2. A test specimen is subjected to a bending moment by weights hung at the end of a lever arm. Torque is applied through the Infinit-Indexer which rotates shaft A with respect to shaft B and holds the relative position of the shafts. Two four-arm strain gage bridges are mounted on the toolholder in the positions shown in Figs. 3 and 4. The output of one bridge is proportional to the strain resulting from the alternating bending stress, and the output of the other bridge is proportional to the shear strain resulting from the steady torque. The numerical relationship between the strains measured at the toolholder and the nominal stress in the specimen groove is established through the calibration program discussed by Kececioglu and McConnell [4].

Two types of tests are conducted: One to determine the cycles-to-failure distribution at a given alternating bending stress level and bending to shear stress ratio. The other to determine the endurance strength, or stress-to-failure, distribution. The steps involved in each type of test are summarized in Fig. 5. A bending stress level and a stress ratio are selected from the overall test program and assigned to one of the three research machines. A PDP-8 computer program* is run to determine the corresponding shear stress and the number of Visicorder divisions required to represent the bending and shear stresses. The Visicorder records the amplified outputs of the strain gage bridges.

* See Appendix A.

The test sequence begins with the installation of a specimen in the toolholder collets, with the groove centered between the collets. With the collet on the strain gage side tightened, the instrumentation is zeroed and calibrated. After tightening the other collet, weights are added to the bending-load arm, and the torque is applied to obtain the desired stresses as indicated by the number of divisions on the Visicorder. At this time a microswitch, which stops the machine when the specimen fails, is set and an interconnected clock is set to zero. The machine is then started, and run at constant speed until the specimen fails. The time to failure is recorded and subsequently used to calculate the number of cycles to failure for that specimen. After running 35 specimens at the specified nominal bending stress level and stress ratio, a data deck is prepared for program STRESS* to be run on a CDC-6400 Computer. The program inputs include the time to failure, Visicorder resistances and divisions used for its calibration, and the divisions recorded during each specimen for the bending and shear stress levels. The program calculates the achieved bending and shear stresses, stress ratio, and cycles to failure for each one of 35 specimens used in each test series. Then it calculates the statistical mean and standard deviation of the bending stress, shear stress, and stress ratio achieved in each test series of 35 specimens.

The cycles to failure of each specimen becomes the input into program CYTOFR** for analysis of the statistical distribution of the 35 cycles-to-failure data. The normal and lognormal distribution parameters are determined, as well as the skewness, kurtosis and the Chi-Squared and Kolmogorov-Smirnov goodness-of-fit test results. The lognormal distribution parameters are then used to construct the distributional S-N diagrams.

* See Appendix B.
** See Appendix C.

The Weibull distribution having gained favor recently in fatigue studies, it was decided to determine the parameters of the Weibull distribution that represents the cycles-to-failure data. PROGRAM WEIBULL* is now used to accomplish this and to see which one of the three distributions (normal, lognormal, and Weibull) represents the cycles-to-failure data best.

The staircase method of testing is used to determine the stress-to-failure, or endurance strength, distribution parameters. The endurance strength is taken to be normally distributed, and is plotted along with the cycles-to-failure distributions to complete the distributional S-N diagrams. The staircase results are also used to construct the very valable distributional Goodman diagrams.

The experimental research for this reporting period consisted of completing the test program planned for Phase I, consisting of cycles-to-failure testing at a stress level of 65,000 psi at the stress ratio of 0.44 and endurance strength testing, to complete the S-N and Goodman diagrams for Phase I research.

In addition the Phase II research was undertaken. The experimental research of Phase II was a continuation of the research performed in Phase I, but with new test specimens.

The Phase I specimens, shown in Fig. 6 were shafts made of AISI 4340 steel, MIL-S-5000 B, Condition C-4, Rockwell C 35/40, grooved to provide a theoretical stress concentration factor of 1.42. The specimens for Phase II, shown in Fig. 7, are identical to the Phase I specimens except that they have a different groove geometry to provide

* See Appendix D.

a theoretical stress concentration factor of 2.34. The results of this research are presented here and compared.

Much valuable fatigue reliability design data has thus been generated experimentally in support of the theoretical methodology for designing specified reliabilities into rotating components subjected to combined reversed bending and steady torque.

On the theoretical side a methodology for generating finite life distributional Goodman diagrams was developed. In addition a promising method to calculate the reliability of rotating components subjected to cumulative fatigue loads was developed and is presented here.

2. EXPERIMENTAL RESEARCH

2.1 Phase I Research Completion.

The results of Phase I experimental research accomplished prior to this period were reported by Kececioglu and Smith [5]. They included endurance tests at stress ratios of ∞, 3.5 and 0.83. Cycles-to-failure tests were accomplished at two alternating stress levels for the stress ratio of 0.44: 69,000 psi and 60,000 psi. The endurance run for the stress ratio of 0.44 was also begun during the previous reporting period but was not completed.

Another cycles-to-failure test series for the stress ratio of 0.44 with eighteen (18) specimens was run on Machine No. 1 at a nominal bending stress level of 65,000 psi. Program STRESS was run on the CDC-6400 Computer to determine the nominal bending and torque stresses in the groove of the specimen and the resulting stress ratio achieved in these tests. The results are summarized in Table 1. From Table 1 it can be seen that the achieved standard deviations are approximately two percent of the mean values of the stresses and the stress ratio. The run was, therefore, under control, and the test results were considered acceptable.

The cycles-to-failure calculated by program STRESS for each specimen were used as inputs into program CYTOFR for the analysis of the distribution of the data. A summary of the outputs from the program is shown in Table 2.

A comparison of the goodness-of-fit results obtained for the normal and lognormal distributions shows the following:

1. The K-S test does not reject either distribution since the maximum D value is less than the allowable in each case.
2. The coefficient of skewness is significantly larger than zero for the log cycles than for the straight cycles; where the coefficient of skewness of the symmetrical normal distribution is zero.
3. The coefficient of kurtosis for the data fitted to a normal distribution is smaller than the value of 3.0 for the normal distribution, and the value obtained for the lognormal distribution is greater than 3.0.

Based on the overall statistics and the conformance of previous results, the lognormal distribution was chosen. The results are plotted in the S-N diagram of Fig. 8. Figs. 9, 10, and 11 give the S-N diagrams for the previous research results of purposes of completeness and comparison. Table 3 summarizes the results used to obtain Figs. 8 thru 11. Table 4 summarizes the results of the normal distribution fit to all Phase I cycles-to-failure data and Table 5 of the lognormal distribution fit, for purposes of completeness and comparison.

Tests to determine the endurance strength for the stress ratio of 0.44* were run on Machine No. 2. The staircase method [5, p. 19] was used with specimens tested to 2.5×10^6 cycles for success.

---

* The stress ratios actually achieved in these tests averaged out to $r_s = 0.45$, consequently these endurance strength results are reported as being for $r_s = 0.45$.

The results for 37 valid data points are shown in Fig. 12. Using the equations presented in Mood and Dixon [8, p. 114] the mean and standard deviation of the endurance strength distribution were calculated, as shown in Table 6, giving an endurance strength mean of 49,600 psi and a standard deviation of 3,700 psi. The results obtained from the cycles-to-failure tests and the endurance tests were used to complete the S-N diagram shown in Fig. 8.

The endurance strength distribution parameters were applied to equation [5, p. 37].

$$\bar{S}_r = \bar{S}_a \left(1 + \frac{1}{\bar{r}_s^{\,2}}\right)^{1/2}, \tag{1}$$

and

$$\sigma_{S_r} = \sigma_{S_a} \left(1 + \frac{1}{\bar{r}_s^{\,2}}\right)^{1/2}, \tag{2}$$

to obtain the parameters of the distribution along $r_s = 0.45$, with the following results:

$$\bar{S}_r = 120{,}900 \text{ psi},$$

and

$$\sigma_{S_r} = 9{,}000 \text{ psi}.$$

The incorporation of this $r_s = 0.45$ strength distribution into the Goodman diagram shown in Fig. 13 completed the distributional Goodman diagram for 2.5×10^6 cycles of life and the Phase I experimental research program. Figures 14, 15, and 16 give the staircase test results for $r_s = \infty$, 3.5, and 1.0 respectively; and Tables 7, 8, and 9 give the corresponding calculations of the endurance strength distribution parameters for purposes of completeness and comparison.

Table 10 summarizes the endurance strength results used to prepare Fig. 13.

2.2 Phase II Research

The Phase II experimental research objective was to obtain distributional S-N and Goodman diagram data with specimens of the same steel as for Phase I but having a theoretical stress concentration factor of 2.34 instead of 1.42 for Phase I.

2.2.1 Geometry and Hardness of Research Specimens

Fig. 7 shows the geometry of the new specimens. The manufacturing processes were carefully controlled, to assure metallurgical and strength similarity to Phase I specimens. Accordingly thirty-five (35) 9" long ungrooved specimens and thirty-five (35) 9" long grooved specimens for use in tensile tests, and one-thousand-fifty (1,050) 6" long specimens for use in fatigue testing were obtained.

Measurements were made of the ungrooved and grooved specimens to verify the contractor's ability to meet machining tolerance and hardness requirements. The first set of tensile test specimens were not acceptable when the hardness and base diameters were found not to be within specifications, and the yield and ultimate strengths of the ungrooved specimens were significantly lower than those for the Phase I specimens.

To assure that Phase I and Phase II specimens, made of two different lots of AISI 4340 steel, were similar except for the change in groove size and its effect on strength, a second set of

tensile test specimens was obtained. The specimen diameters and radii were measured with an Optical Comparator, at 20 magnification, located at the Arizona Gear Co. in Tucson. The results are given in Tables 11 and 12. The surface finish at the base of the groove of the grooved specimens was observed through a microscope and visually compared with Johansen's precision gage blocks made by Pratt and Whitney because of the inaccessibility of the base of the groove to a Profilometer. Surface hardness measurements were made in the University's Metallurgival Laboratory with a Wilson Rockwell C Hardness Tester using a 150-K_g load and the Braile indenter. The results are given in Tables 11 and 12. In addition, hardness tests were made on interior cross sectional areas of three grooved and three ungrooved specimens to determine the uniformity of hardness throughout each specimen. Standard precautions were taken to assure that the original hardness was not altered during sectioning. The location of test sites for the interior hardness measurements are shown in Figs. 17 and 18. The results of the interior hardness measurements are given on Tables 13 and 14.

Applying a 3σ analysis to the surface hardness data in Tables 11 and 12 indicates that 99.73% of the population would lie between the limits of 36 R_c and 39 R_c for the grooved specimens, and between 35 R_c and 39 R_c for the ungrooved specimens, hence within the specifications of R_c 35/40. The hardness readings in the sections given in Tables 13 and 14, show a preponderance of values between 35.5 R_c and 37.5 R_c with a high degree of uniformity. Since the hardness at the surface and in the interior of the specimens met the

specification requirements of R_c 35/40, it was concluded that the specimens were given a proper heat treatment.

2.2.2 Strength Characteristics of Research Specimens

The ungrooved and grooved specimens were subjected to tensile loads to determine their yield, ultimate and breaking strengths. The tensile pull tests were performed at the Hughes Aircraft Company, Tucson, Arizona on a 60,000 lb. Tinius Olsen test machine. The machine was calibrated a short time before the tests and was considered to be in a fully operational condition.

The thirty-five (35) ungrooved specimens were tested with an extensometer attached to each specimen which provided elongation input to a load-elongation pen recorder. The load was concurrently displayed on a 30-in. diameter dial segmented into to psi intervals on the 0 - 60,000 psi scale. The elongation of a two-inch gage length was measured with a micrometer after the specimen failed, and the percent elongation was calculated.

As the specimen was placed under tension at a constant rate of elongation, observers watched the dial in an effort to identify the yield, ultimate and breaking loads. Upon reaching the ultimate point, the specimen was unloaded, the extensometer was removed, and the specimen diameter was measured with a micrometer to determine any reduction in cross-sectional area. The load was reapplied and the specimen was stressed to the breaking point. The yield load was identified on the recording at the 2% elongation point and the ultimate load was identified by the maximum load recorded on the chart. The

load at the breaking point, determined by visually observing the moving pointer, was manually recorded. The yield, ultimate, and breaking loads thus obtained are given in Table 15 and the percent elongation data and results are given in Table 16.

After completion of the tensile test, the diameter of the ungrooved specimens were remeasured on the Optical Comparator to determine the area to be used in calculating the breaking strength. The final diameters and the elongation measurements are given in Tables 15 and 16. The yield and ultimate strengths were calculated using the original (pre-test) specimen diameters; whereas, the breaking strength was calculated on the basis of the final (reduced by elongation) diameters. The results are given on Table 15 and are summarized at the end of Table 15.

During the testing of the grooved specimens the extensometer could not be used and the dial readings were visually observed and manually recorded. During the test of the first specimen it was observed that (1) the yield load could not be positively identified, (2) there was little time lag and reduction in load between the ultimate and the breaking loads, and (3) the actual fracture of the specimen was accompanied by a loud shock wave throughout the testing laboratory. A decision was made thereafter to unload the specimens after the ultimate load was reached. Thus only the ultimate loads could be obtained, and are given in Table 17. Any change in specimen diameter at the ultimate load could not be measured accurately enough, hence it was decided to use the original (pre-test) area to calculate the

ultimate strength. The results are given in Table 18, and a summary thereof is given at the end of Table 17.

2.2.3 Analysis of Tensile Test Results

Before making the final decision regarding the acceptability of the tensile test specimens as a basis for procuring fatigue test specimens, a review was made of the strength parameters obtained from test of the Phase I and Phase II specimens. The strength parameters of the Phase I tensile test specimens, extracted from our previous report [3, Tables 7 and 8], and the results of the Phase II tests just discussed are listed in Table 18. It is noted that the mean yield and ultimate strengths obtained from Phase II tensile tests of ungrooved specimens are lower than the yield and ultimate strengths of the Phase I specimens. The standard deviations are different but within 4 percent of the respective means. The ultimate strength for the grooved specimens of Phase II was higher than for Phase I. This should be expected because Phase II specimens have a smaller groove radius which results in a higher static ultimate strength; consequently, this does not provide a basis for believing that the Phase II and Phase I specimens are different.

The student "t" test was used to perform a comparison of the sample means of the ultimate strengths of the Phase I and Phase II tensile test specimens [7, pp. 193-194]. This method was selected because it is widely accepted to be valid for small sample sizes, and the Phase I data means were based on a sample size of 10. The results of the statistical tests and the corresponding critical

values at the 0.05 level of significance are as follows:

	t - Statistic	t - Statistic Critical Value
Ungrooved Specimens	19.65	2.02
Grooved Specimens	13.10	2.02

The t-statistic is larger than the t-critical value which indicates that the difference between the means of the ultimate strength is statistically significant. The Phase I ungrooved specimens are apparently the stronger.

The sample variances of the ultimate strengths were compared using the F-test [8, pp. 167-172]. The results, at the 0.05 level of significance, are as follows:

	F - Statistic	F - Statistic Critical Value
Ungrooved Specimens	3.00	2.84
Grooved Specimens	1.08	2.85

The F-test for the variance further indicated a significant difference between the Phase I and Phase II ungrooved specimens but not between the grooved specimens.

The results of the statistical analysis do not provide a logical basis for accepting or rejecting the Phase II specimens. Thus it became necessary to use different criteria to determine the acceptability of the tensile test specimens as a basis for ordering fatigue test specimens.

A metallurgical analysis of a specimen was obtained to determine its composition as AISI 4340 steel and its compliance with MIL-S-5000B. The analysis performed by Magnaflux Corporation, Materials Testing Laboratories confirmed that the sample met all requirements for the chemical composition.

A review of the data from the physical measurements given in Tables 11 thru 14 confirmed that the test specimens met all specification requirements. The hardness values, which are considered to be most critical are well within specification requirements for surface hardness, and the additional hardness measurements on interior cross sectional surfaces also confirmed the uniformity of the hardness. All factors considered, the decision was made to proceed with the procurement of fatigue test specimens for Phase II.

2.2.4 Recalibration of NASA Complex-Fatigue Research Machines

2.2.4.1 Requirements

Machine No. 2 required recalibration since it had been modified by the installation of spherical bearings at the Flex Couplings. Machines 1 and 3 were also recalibrated so that the strain gages of all machines would be verified to be functioning properly, and to revise the calibration coefficients if the calibration results so indicated. The speed of each machine was also measured to assure its constancy and to determine if any change occurred due to wear in each machine and its drive motor.

2.2.4.2 Procedures and Results

The calibration method and procedures used were the same as those described in NASA CR - 72839 [4, pp. 30-33]. The bending calibration was accomplished in two phases. First an out-of-machine bending calibration was done using the setup shown in Fig. 19. This phase verified that the bending bridge strain gages, shown in Fig. 3, mounted on the toolholder arm of each machine and the strain gage mounted in the groove of the calibration specimen were functioning properly and provided a relationship between the actual and the apparent strain in the toolholder strain gages. This was based on the facts that the actual/apparent stress ratio was close to the one it should be, and the plot of strain from the gage bridge versus applied weight was linear.

The test setup shown in Fig. 20 was used to determine the calibration coefficients for the torque bridge, shown in Fig. 4, and for any interaction between torque and bending bridges.

Next an in-machine, quasi-dynamic calibration was performed which provided the calibration coefficient needed to calculate nominal bending stress in the specimen groove from the apparent stress at the toolholder.

Each machine was carefully calibrated with observations repeated a minimum of six times at each test point. The relation between the calibration variables, in each case, proved to be linear with the functional relationship

beginning at the origin. Thus the slope of each regression line for the calibration data completely defined the function and provided the calibration coefficients listed in Table 19.

The rotating speed of each machine was determined using a tachometer strobe light which contains an internal oscillator and 60-cycle calibration. The results confirmed proper operation of each induction motor in providing a constant speed drive to each research machine. It was also found that the speed of each machine was independent of the bending and torque loads. The speed of each machine is listed in Table 19. The last calibration coefficients of the previous calibration was designated as Mode 4. The coefficients designated as Mode 5 apply to all data from June 1, 1971 until the next calibration.

2.2.5 Ungrooved Specimens Fatigue Research

Thirty-five Phase I specimens were machined down as shown in Fig. 21 so as not to provide any stress concentration at all. These specimens were tested in the Ann Arbor (R. R. Moore type) rotating beam fatigue research machine in our Reliability Research Laboratory to determine the endurance strength of ungrooved specimens.

An optical comparator was used to determine the point of minimum diameter of each specimen and to measure the diameter. It was found that the specimens had a mean diameter of 0.3151 in., a standard deviation of 0.0004 in., and a range of 0.3143 in. to 0.3159 in.

A computer program was written and executed on the PDP-8 Computer to calculate the bending moments and pan loads required to test ungrooved specimens at target stress levels and desired specimens diameters.* The program was based on the following calibration equation for the Ann Arbor machine:

$$M = 0.267 + 4.09L \tag{3}$$

where

M = bending moment at the test section

L = total effective load = pan weight (P) + 8.625 lb.

Therefore the pan weight, P, to obtain a bending moment, M, is given by

$$P = \frac{(M + 0.267)}{4.09} - 8.625 \tag{4}$$

where

$$M = \frac{\pi D^3 \times \text{Stress}}{32}, \tag{5}$$

and D is the test section diameter of the specimen to be tested next.

In view of the range of specimen diameters, the computer program was run to calculate pan loads at diameter intervals of 0.0005 in. from 0.3140 in. to 0.3160 in. for each stress level desired. With interpolation, the required pan weights could be determined to the nearest $\pm$ 0.10 lb.

The staircase method of determining the endurance strength was used with a stress increment of 1,900 psi and a life of 2.5×10^6 cycles. A specimen was selected at random, its diameter obtained from the table of specimen diameters, and the appropriate pan

* This program is given in Appendix F.

weight as determined from the PDP program printout was used to apply the desired stress level.

Thirty-eight valid runs were completed, as shown in the staircase plot of Fig. 22, with 15 successes and 13 failures. These data were then used to calculate the mean and standard deviation of the endurance strength distribution at 2.5 x 10^6 cycles of life for the AISI 4340 steel, R_c 35/40, ungrooved specimens subjected to an alternating bending stress and a constant shear stress with a shear ratio of $r_s = \infty$, as shown in Table 20. The mean endurance strength was found to be 80,725 psi and the standard deviation 3,040 psi. In comparison, the published endurance strength is estimated to be approximately 89,000 psi for polished specimens and 81,000 psi for machined specimens [9, pp. 160-172]. The mean endurance strength as determined by this test program is very close to the published endurance strength of polished specimens. Thus the results of the tests are reasonable and compatible with current fatigue failure theory.

When the decision was made to procure a second group of tensile test specimens, it was decided to obtain another group of ungrooved specimens and repeat the above tests to provide another basis for accepting the Phase II specimens as having physical properties close to those of Phase I specimens. A number of Ann Arbor research machine outages had been encountered because the bending moments at higher stress levels approached the limit of the machine. Thus, it was decided to reduce the diameters of the new specimens to a nominal value of 0.2500 in. Thiry-five Phase II research specimens were obtained as per Fig. 21. The diameter of the specimens were

measured at the minimum point and the mean diameter was found to be 0.2504 in. with a standard deviation of 0.0004 in. A specimen was randomly selected and the appropriate pan weight was determined from the PDP Computer program as before. The endurance life was again taken to be 2.5×10^6 cycles and the staircase stress increment 1,900 psi.

Thirty-four useful data points consisting of 18 successes and 16 failures were obtained, as shown in the staircase plot of Fig. 23. The 16 failures were used in the calculations given in Table 21. The endurance strength distribution parameters of the 0.2500 diameter, ungrooved, Phase II research specimens subjected to an alternating bending stress and no shear stress, with a stress ratio $r_s = \infty$, were found to be: Mean = 80,230 psi, and standard deviation = 1,425 psi. As the mean endurance strength was identical for both Phase I and Phase II steel specimens, the decision to continue using this steel and the manufacturer of the research specimens for the Phase II research was upheld.

2.2.6 Grooved Specimens Fatigue Research

2.2.6.1 Cycles-to-Failure Tests

The Phase II test program using the new specimens, grooved to provide a theoretical stress concentration factor of 2.34, was initiated at the stress ratio of infinity. Using the von Mises-Hencky failure theory, the stress ratio , r_s, for the loading provided by the research machines used in this research, is defined as

$$r_s = \frac{S_a}{\sqrt{3}\ \tau_{xym}} \qquad (8)$$

where

S_a = alternating reversed bending stress due to the bending moment.

τ_{xym} = mean shear stress due to torque.

Cycles-to-failure tests at the stress ratio of infinity were conducted at mean nominal alternating stress levels of 108,900, 92,100, 73,600, 49,400, and 39,300 psi. Five stress levels are chosen to obtain data over the finite life range for the preparation of the corresponding S-N diagram.

Upon completion of testing at the stress ratio of infinity, a decision was made to conduct all fatigue tests at a specific ratio on one test machine. Therefore, tests were initiated and completed at stress ratios of 1.06, 0.40, 0.25, and 0.15. Fewer stress levels were run as shown in Table 19 for stress ratios lower than ∞ because of yield stress limitations in shear.

The sample size to obtain the cycles-to-failure distribution for each alternating stress level and stress ratio combination was increased to 35, as recommended in the previous report [5, p. 95]. The actual alternating bending stress, the shear stress, the normal mean stress, and the stress ratio for each specimen in the sample were calculated by computer program STRESS. In addition, the mean and standard deviation of the achieved stresses for each sample were calculated, and are given in Table 22.

Program STRESS was updated for Phase II to incorporate current calibration constants for each machine to reduce the test data, and add rpm values for each machine. The computer printout includes a listing of the cycles to failure for each specimen in the sample. The updated program in Fortran language is given in Appendix B.

Individual cycles-to-failure data were used as input data for program CYTOFR, as discussed in the previous report to NASA [5], to calculate the cycles-to-failure distribution parameters for the normal and $\log_e$ normal distributions and perform goodness-of-fit tests. Program CYTOFR was further updated to incorporate Cal-Comp graph and plot subroutines. Subroutine GRAPH constructs a histogram of the cycles to failure based on the failures per cell determined by the Chi-Squared test, and superimposes the distribution curve from the parameters computed by the main CYTOFR program on each histogram. The Chi-Squared test modification incorporated the automatic combining of cells at the tails of the distribution when the end cells do not contain at least five failure data points. The updated program in extended Fortran language is given in Appendix C.

Program CYTOFR was run for each alternating bending stress level at which specimens were tested at the stress ratios of infinity, 1.06, 0.40, 0.25 and 0.15. Typical examples of the histograms and curves for the normal and $\log_e$ normal distributions are given in Figs. 24 and 25. The computed distribution parameters and values of the goodness-of-fit tests are summarized in Table 23 for the normal distribution, and in Table 24 for the $\log_e$ normal distribution.

The goodness-of-fit test parameters listed in these tables were reviewed to determine if the applicable distribution of the cycles-to-failure data for Phase II specimens differed significantly from that for Phase I specimens. The following observations were made:

1. The values of the coefficients of skewness and kurtosis for the normal and the lognormal distributions were approximately the same and provided no preference for either distribution.
2. The K-S goodness-of-fit test does not reject either distribution at the 0.05 level of significance. The largest D value was 0.201 for the normal distribution and 0.160 for the lognormal distribution; both were less then the critical D value of 0.224 at the 0.05 level of significance for a sample size of 35. Furthermore, there was essentially no difference between the normal and lognormal D values at any stress level.
3. The Chi-Squared test proved to be more discriminating than the K-S test. The Chi-Squared value and the appropriate degrees of freedom (d.o.f.) for each stress level were compared with the critical value of 3.841 for 1 d.o.f. and of 5.991 for 2 d.o.f. . The normal distribution was rejected in three out of seventeen samples; and the lognormal distribution was rejected in only two out of seventeen samples. One test each for normality and lognormality was inapplicable because

the data points were contained in only three cells resulting in zero degrees of freedom.

4. It was concluded from these observations that there is a preference for the lognormal distribution over the normal distribution when working with cycles-to-failure data.

The $\log_e$ normal distribution parameters from Table 24 were used to construct the S-N diagrams for the Phase II specimens. These S-N diagrams are given in Figs. 26 thru 30.

2.2.6.2 Endurance Tests

Endurance runs for Phase II specimens were completed using the "staircase" method at alternating bending to mean shear stress ratios of ∞, 1.06, 0.40, 0.25 and 0.15. The staircase plots are given in Figs. 31 thru 35. The endurance strength distribution parameters were calculated in Tables 25 thru 29, and are summarized in Table 30. These parameters were used to complete the S-N diagrams of Figs. 26 thru 30, and to construct the distributional Goodman diagram of Fig. 36.

3. THEORETICAL RESEARCH

3.1 Generation of Finite Life Distributional Goodman Diagrams for Reliability Prediction

A methodology for developing finite life distributional Goodman strength surfaces from cycles-to-failure distributions at specified alternating stress levels has been developed by Kececioglu and Guerrieri [10]. The Goodman strength surface shows the combinations of alternating bending stress and mean shear stress allowable to the design engineer. This study also investigated the applicability of the distortion energy and the maximum shear stress failure theories to determine which provided better correlation with the experimental data generated during Phase I. The finite life Goodman surface, developed using from two to five calculated strength distributions at specified stress ratios, can be used to construct distributions at any desired stress ratio and applied to probabilistic design.

It is also found that the von Mises-Hencky ellipse effectively models Goodman diagrams for life greater than 10,000 cycles. However, when the equation for the von Mises-Hencky ellipse was modified from

$$\left(S_a/S_e\right)^2 + \left(S_m/S_u\right)^2 = 1 \tag{9}$$

to

$$\left(S_a/S_e\right)^a + \left(S_m/S_u\right)^2 = 1 \tag{10}$$

and the values of a were determined from plots of finite life Goodman diagrams, the values ranged from 1.91 at 200,000 cycyles to 2.38 at

40,000 cycles. This is an area that requires further study in search of a general equation, or applicable values of a, valid over all ranges of cycles to failure.

3.2 The Weibull Distribution as A Description of Fatigue Life

3.2.1. Introduction

Since the Weibull distribution was introduced in 1949, it has gained wide acceptance as an extreme value distribution [11]. It has been used extensively in such areas as bearing fatigue data analysis [12], and the prediction of the failure of automotive parts.

The Cal-Comp plots of normal and lognormal distributions did not fit the cycles-to-failure data very well for some sets of data. It was suggested that the three-parameter Weibull distribution may provide a better fit.

The theory of the Weibull distribution and its manual application to experimental data are described in the existing literature [11], [12]. This study developed and validated a computer program which computes the parameters of the three-parameter Weibull distribution for a set of cycles-to-failure data, performs goodness-of-fit tests, and calculates the cycles to failure for 0.90 and 0.99 reliability with 90% confidence. The program was tested using the data generated under Phase II and the results were analyzed.

3.2.2 Development

The computer program development was patterned after the method described by Lochner [12], which used Weibull probability paper and manual calculations. The foundation is the generalized Weibull frequency distribution shown in Hahn and Shapiro [11, p. 110]

$$f(N;\ \beta,\ \eta,\ \gamma) = \frac{\beta}{\eta}\left(\frac{N-\gamma}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{N-\gamma}{\eta}\right)^{\beta}\right], \tag{11}$$

$$N > \gamma,\ -\infty < \gamma < o,\ \beta > o,\ \eta > o,$$

where

N = cycles to failure

β = shape parameter or Weibull slope

η = scale parameter

γ = location parameter .

From the basic definition of reliability

$$R = \int_N^{\infty} f(N)\ dN,$$

the relationship between reliability and fatigue life N is

$$R = e^{-\left(\frac{N-\gamma}{\eta}\right)^{\beta}}, \tag{12}$$

where γ, η, and β are constants to be determined by the analysis of test data. The fraction failed, or unreliability Q, for N cycles is given by

$$Q = 1 - e^{-\left(\frac{N - \gamma}{\eta}\right)^{\beta}} , \tag{13}$$

Equation (13) gives the probability of failure in N cycles or less, and is the cumulative distribution function, F(N). It can be transformed into a linear form by taking natural logarithms as follows:

$$F(N) = 1 - e^{-\left(\frac{N - \gamma}{\eta}\right)^{\beta}},$$

$$1 - F(N) = e^{-\left(\frac{N - \gamma}{\eta}\right)^{\beta}},$$

and

$$\ln \ln \left[\frac{1}{1 - F(N)}\right] = \beta \ln (N - \gamma) - \beta \ln \eta. \tag{14}$$

Letting

$$y = \ln \ln \left[\frac{1}{1 - F(N)}\right] , \tag{15}$$

and

$$x = \ln (N - \gamma), \tag{16}$$

Eq. 14 becomes

$$y = \beta x + \text{constant},$$

or a transformed linear function.

Weibull probability paper has been prepared with log log versus log scales so that the plot of y versus x of data would be a straight line with a slope of β. When $(N-\gamma) = \eta$, $F(t) = 1 - e^{-\left(\frac{\eta}{\eta}\right)^{\beta}} = 1 - e^{-1} = 0.632$, thus the value of $(N-\hat{\gamma})$ at which $\hat{F}(N) = 0.632$ is an estimate of η. The location, parameter, γ, is the minimum life point that provides the best approximation to linearity between x and y.

3.2.3 Weibull Computer Program

The basic FORTRAN computer program to determine the estimates of the Weibull parameters for cycles of life for specified levels of reliability was provided by Mr. Thomas C. Stansberry, Delco Radio Division, General Motors Corporation. His program was adapted to The University of Arizona CDC 6400 Computer and was updated to include subroutines for the Chi-Squared and Kolmogorov-Smirnov goodness-of-fit tests. Program WEIBULL is given in Appendix D.

The first data card contains the sample size and the minimum life increment for use in linearizing the x-y relationship. Subsequent data cards (one for each specimen) contain the cycles-to-failure information. The first operation performed by the computer is to establish an ordered array of the cycles to failure and the corresponding median ranks. The computer calculates the median ranks, $y_i = \ln \ln \left(\frac{1}{1 - F(N_i)}\right)$, and $x_i = \ln (N_i - \gamma_k)$. Where i = 1, 2, ---, n and γ_k = minimum life increment (1, 2, ---, k) such that $\gamma_k < N_1$. As the array of y_i and x_i is computed for different γ_k the method of least squares is used to determine the degree of linearity. This operation is iterated with γ_k being increased in increments until the best fit straight line is obtained. At that time the computer records the estimates of γ, β, and η. It then calculates the one percent failing life, the ten percent failing life, and the 50 percent failing life with the associated 90 percent confidence limits.

Upon completion of the calculations, the program calls the K-S and Chi-Squared test subroutines, in turn, to provide a measure of goodness of the fit of the estimated Weibull distribution

with the data. The K-S subroutine, "DTEST", applies the Kolmogorov-Smirnov goodness-of-fit test and prints differences, D, for each failure time in ascending order. Analysis of the K-S test is done by comparing the largest in absolute D value, with its critical value obtained from a D value table.

The Chi-Squared test requires the subdivision of the cycles-to-failure data into a number of cells, k, determined by Sturges' rule [13]

$$k = 1 + 3.3 \log_{10}(n) , \tag{17}$$

where n is the sample size of the data. However, analysis of the results requires at least five data points in each cell. The use of Sturges' rule for data with a sample size of 35, results in six cells of equal width. Consequently, when the data is grouped into these six cells the cells at each end usually end up with fewer than five data points. If the two end cells are combined to provide five or more data points, the number of filled cells reduce to as few as four. Since the distribution being tested is the three parameter (r=3) Weibull, the degrees of freedom (k-r-1) requires that the number of cells, k, be at least five in order to have at least one degree of freedom. This Chi-Squared test was applied to samples of Phase II cycles-to-failure data. It was found that six of the twelve tests resulted in only four filled cells. Thus, there was zero degrees of freedom and the Chi-Squared test was not useable. Thus, it appears that the sample size will have to be increased still further if the standard Chi-Squared goodness-of-fit test is to be used.

To circumvent this problem variable cell widths were used. The technique described by Hahn and Shapiro [11, pp. 302-308] called for the calculation of cell widths to provide equal number of observations in each cell. A modification to the subroutine was made dividing the range of the data so that each cell contains exactly five cycles-to-failure data. For our sample size of 35, this provides seven cells. This subroutine was run for the same 12 sets of data. The expected frequency for the seventh cell was always less than five, which according to accepted practices invalidate the test. It was observed, however, that as long as the expected frequency was equal to or greater than two, the Chi-Squared value could be calculated. This observation was confirmed by a Monte Carlo Simulation of 1,000 runs from which it was concluded that Chi-Squared errors resulting from expected frequencies between two and five are insignificant. Nevertheless, a final modification was made to the subroutine using variable cell widths, but combining adjoining end cells to insure that the expected number of observations per cell is equal to or greater than five. When the same cycles-to-failure data was rerun to apply this Chi-Squared test subroutine, all 12 tests resulted in six useable cells thus providing two degrees of freedom.

3.2.4 Results

The operation and accuracy of the computer program were verified by using the same input data used by Lochner [12]. Identical estimates were obtained for each parameter to the degree of accuracy obtainable from probability paper plots. The computer program

provided parameter estimates to five place accuracy and used this accuracy in subsequent calculations. In Table 31 the Weibull distribution parameters, and the K-S and Chi-Squared goodness-of-fit results are given. A sample Cal-Comp plot of the Weibull distribution is given in Fig. 37.

The accuracy of the D-test subroutine for the Kolmogorov-Smirnov goodness-of-fit test was confirmed by a desk calculator. The maximum D values found by applying the subroutines to the cycles-to-failure data are listed in Table 31. These results show that, at the 0.05 level of significance, the K-S test does not reject the Weibull distribution in all of the 17 tests. Thus, the Weibull distribution can safely be used to approximate distributions of cycles-to-failure data at specified stress levels.

The Chi-Squared test values determined by the WEIBULL subroutines with variable cell widths are also given in Table 31. Note that the variable cell width analysis rejects 5 out of 17 tests.

The conclusions drawn from the above analysis are listed below:

1. Based on the K-S test results of not rejecting any of the samples, the three-parameter Weibull distribution may describe fatigue cycles-to-failure data.
2. Based on the Chi-Squared test results of 5 rejections out of 17 samples, the Weibull distribution may not be considered generally acceptable for cycles-to-failure data consisting of 35 data points. Further in 14 out 16 cases

the Chi-Squared value for the Weibull is greater than for the lognormal.

3. Based on the previous two conclusions and the results in Tables 24 and 31, the lognormal distribution appears to represent the cycles-to-failure data of the Phase II research best.

3.3 Reliability of Components Subjected to Cumulative Fatigue

3.3.1 Introduction

In cumulative fatigue of most concern to design engineers is the mathematical relationship between the number of load cycles at various alternating stress levels applied to a component, the S-N diagram results, and the survival life of the component under these conditions. After such a relationship is determined a method needs to be developed to predict the reliability of a component subjected to a specified history of cumulative fatigue stresses. The objective of this study was to review the published cumulative fatigue theories, and to discover or come up with methods for making reliability predictions.

3.3.2 Literature Search

The literature search revealed attempts to describe the degree of cumulative damage in expressions involving transfer of energy or mass, with damage often described by a crack parameter and interpreted by Osgood [14] as a change in the state of energy in the immediately adjacent volumes of material. The primary difficulty with these methods is their complexity and highly approximate nature.

The simplest and most widely used cumulative damage rule is Miner's rule [15], based on the assumption that cumulative damage under cyclic stressing is related to the net work absorbed by the specimen. That is, the number of stress cycles applied, expressed as a percentage of the number of cycles of life at the given alternating stress level, is the proportion of useful life expended. Therefore, the specimen should fail when the total damage reaches 1.00, or

$$\sum_{i=1}^{m} \frac{n_i}{N_i} = 1 \tag{18}$$

where

$n_1, n_2, \ldots, n_m$ = cycles of operation at each applied alternating stress level.

$N_1, N_2 \ldots, N_m$ = cycles of life at each stress level. Miner's

rule is accepted as providing a good, conservative first approximation for engineers in preliminary design, but fails to account for the effects of overstressing or understressing in the early cycles or for loading sequence.

An expression was developed by Corten and Dolan [16] to model the hypothesis that fatigue damage in terms of the nucleation of submicroscopic voids which develop into cracks, is a function of damage nuclei and the rate of damage propagation. Damage, which was represented as a power function of the number of cycles, was summed for a loading sequence consisting of repeated blocks of cycles alternating between two stress amplitudes. The functional relationship developed is

$$N_g = \frac{N_1}{\alpha_1 + \alpha_2 (\frac{S_2}{S_1})^d + \alpha_3 (\frac{S_3}{S_1})^d + \ldots + \alpha_n (\frac{S_n}{S_1})^d} , \qquad (19)$$

where

N_g = total number of cycles of stress to failure for an incremental stress amplitude history,

N_1 = number of cycles at the highest stress level, S_1, before failure,

$\alpha_1, \alpha_2, \ldots, \alpha_n$ = ratio of the number of cycles applied at stress levels $S_1, S_2, \ldots, S_n$ to the total cycles applied,

$S_1 > S_2 > \ldots > S_n$ = various alternating stress levels or amplitudes applied,

d = inverse slope of the linear portion of the S-N diagram.

The Corten-Dolan method appears to give a better correlation with existing data than Miner's rule; however, it still has the deficiencies that the value of d cannot be determined with a reasonable accuracy, and the equation is based on a deterministic rather than a distributional S-N diagram.

The NERVA program [17] approaches the cumulative fatigue problem by revising Miner's rule as follows:

$$\sum_{i=1}^{m} \frac{n_i}{N_i} = \gamma, \tag{20}$$

where

γ = a normally distributed variable with a mean, $\bar{\gamma}$, and a standard deviation, σ_γ.

The experimental values of γ have been found to range between 0.18 and 23.0, depending upon the material, the test conditions and the order of loading history. It was observed that a low to high loading sequence ($s_1 < s_2 < s_3$..) resulted in high γ values ($\gamma > 1$). A high to low loading sequence ($s_1 > s_2 > s_3$ ...) gave low γ values ($\gamma < 1$). For a loading history with representative high, low, and medium stress levels in random order the value of γ appears to be close to unity.

Sorensen [18] developed a general expression for the probability distribution of the damage rate in terms of the power spectrum of the random excitation. The method requires use of the single valued theoretical S-N diagram to determine distributional values. The analysis is logical and the results

obtained from a numerical example appear to be reasonable. The method warrants further investigation using distributional S-N diagrams developed by this research. Serensen [19, pp. 33-43] studied fatigue damage accumulation under distributional service loading. A family of curves for the probability density function of the stress amplitude, the distribution of fatigue life for a given group of parts at a given type of loading, and Miner's rule are used to assess fatigue damage. The analysis and probabilistic calculations are logical but are subject to the limitations of Miner's rule.

3.3.3 Proposed Methods to Calculating the Reliability of Components Subjected to Cumulative Fatigue at Sequenced Stress Levels.

Use was made of the cumulative fatigue theories and the statistical nature of fatigue life to develop methodologies for the calculation of reliability. Two methods were developed.

The first method makes use of the multiplication rule and conditional reliabilities or probabilities of survival. With this approach the first step is to calculate the probability, P_1 of surviving the first N_1 cycles at stress level S_1. The next step is to compute the probability, P_2, of surviving N_2 cycles at stress level S_2 given survival of N_1 cycles at stress level S_2. Then the probability of survival for the sum of these cycles $(N_1 + N_2)$ is the product of the individual probabilities $(P_1 \cdot P_2)$. This procedure would be continued for as many steps as necessary.

The second method is called the method of equivalent reliabilities. Here the probability of surviving N_1 cycles at stress level S_1 is computed. Utilizing this information an equivalent cycle life, N_1', at stress level S_2 is computed. The N_2 cycles at stress level S_2 are now added to N_1' and the probability of surviving the N_1' + N_2 cycles at stress level S_2 is computed. This value is equivalent to the reliability associated with the survival of the N_1 cycles at stress level S_1 and the N_2 cycles at stress level S_2. Again, this procedure could be continued as many times as needed to obtain the final reliability of a component subjected to cumulative fatigue at sequenced stress levels.

The two methods will be evaluated next using life cycle data obtained from the experimental test program. Consider the following stress history applied to a steel shaft:

Alternating Stress Level	Cycles Run
S_1 = 86,000 psi	10,000
S_2 = 96,000 psi	1,000
S_3 = 100,000 psi	500

The fatigue life of specimens tested at these mean stress levels were found to be as follows:

Alternating Stress Level	Fatigue Life	
	Mean Cycles Log_{10}	Standard Deviation Log_{10} Cycles
S_1 = 86,000 psi	4.715	0.068
S_2 = 96,000 psi	4.394	0.052
S_3 =100,000 psi	4.102	0.073

Consider first the method that uses the multiplication rule and conditional probabilities. With the stress levels increasing, the reliability for the first level is given by

$$R_1 = \int_{N}^{\infty} f_1 \, (N_1) \, dN_1 \; , \tag{21}$$

where $f_1(N_1)$ is the normal probability density function (pdf) for the $\log_{10}$ cycles-to-failure at stress level S_1. Transforming to the standard normal pdf variable

$$z = \left(\frac{N-\bar{N}}{\sigma_N}\right), \text{ gives}$$

$$R_1 = \int_{z_1}^{\infty} \phi \, (z) \, dz \, . \tag{22}$$

In this example

$$z_1 = \frac{\log_{10}(10{,}000) - 4.715}{0.068} = -10.5,$$

which from standard normal distribution area tables yields a reliability, R_1, of essentially 1 from Eqs. (21) and (22).

The conditional reliability for the second stress level is given by

$$R_2 = \frac{\int_{N_2}^{\infty} f_2(N_2)\, dN_2}{\int_{N_1}^{\infty} f_2(N_2)\, dN_2}, \tag{23}$$

where $f_2(N_2)$ is the pdf of cycles-to-failure at stress level S_2. Transforming to the standard normal pdf variables results in

$$R_2 = \frac{\int_{z_2}^{\infty} \phi\,(z)\, dz}{\int_{z_2'}^{\infty} \phi\,(z)\, dz}, \tag{24}$$

where

$$z_2 = \frac{\log_{10}\,(11{,}000)\ -4.394}{0.052} = -6.75,$$

and

$$z_2' = \frac{\log_{10}\,(10.000)\ -4.394}{0.052} = -7.54$$

Insertion of these values of z_2 and z_2' into Eq.(24) yields a reliability, R_2, of 0.9_{10}.*

Continuing in a similar manner for the third stress level, the reliability is given by

* The symbol 0.9_{10} is used to represent the number of 0.9999999999.

$$R_3 = \frac{\int_{N_3}^{\infty} f_3(N_3) \quad dN_3}{\int_{N_2}^{\infty} f_3(N_3) \quad dN_3}, \tag{25}$$

or

$$R_3 = \frac{\int_{z_3}^{\infty} \phi\ (z)\ dz}{\int_{z_3'}^{\infty} \phi\ (z)\ dz}, \tag{26}$$

where

$$z_3 = \frac{\log_{10}(11{,}500)\ -4.102}{0.073},$$

and

$$z_3' = \frac{\log_{10}(11{,}000)\ -4.102}{0.073}.$$

The value for R_3 is found to be 0.9004.

The reliability of the shaft for the mission length of 11,500 cycles and the given alternating stress history is obtained by $R_s = R_1\ R_2\ R_3$, which results in $R_s = 0.90003$.

The reliability was calculated with the stress history reversed; i.e. decreasing stress level. The resulting reliability was 0.9_{21}.

The same problem is approached next utilizing the method of equivalent reliabilities. The increasing stress history is considered first. Calculating the z value for the first stress level gives

$$z_1 = \frac{\log_{10}(10{,}000) - 4.715}{0.068} = -10.5 \quad .$$

Utilizing this value the equivalent number of cycles at stress level, S_2, is found as follows:

$$z_1 = z_2' = -10.5,$$

where

$$z_2' = \frac{\log_{10} N_2' - 4.394}{0.052} \, ,$$

so that

$$\log_{10} N_2' = z_2' \, (0.052) + 4.394.$$

Solving for N_2' results in 7,050 cycles. This means that 7,050 cycles run at stress level S_2 is equivalent to 10,000 cycles at stress level S_1. Thus, to find the reliability associated with both stress histories, add the 1,000 cycles run at stress level S_2 to the equivalent number of cycles and calculate the new z value, or

$$z_2 = \frac{\log_{10}(8{,}050) - 4.394}{0.052} = -9.38 \quad .$$

In like manner the equivalent number of cycles at stress level S_3 is calculated as follows:

$$z_2 = z_3' = -9.38,$$

$$z_3' = \frac{\log_{10} N_3' - 4.102}{0.073},$$

$$\text{Log}_{10} N_3' = z_3' (0.073) + 4.102,$$

or

$$N_3' = 2{,}610.$$

Adding the 500 cycles run at stress level S_3 and computing the z value, the reliability is found to be

$$N_3 = N_3' + 500 = 3{,}110,$$

$$z_3 = \frac{\log_{10} (3{,}110) - 4.102}{0.073} = -8.34,$$

and

$$R = \int_{z_3}^{\infty} f(z)\, dz, \qquad (27)$$

or

$$R = 0.9_{15}\ .$$

The method when applied to the stress history applied in decreasing order yields a z_3 value of -8.36 and a corresponding reliability of 0.9_{15}.

Comparison of the two methods indicates the following: 1. The method utilizing conditional probabilities gives a substantially lower reliability with the increasing stress history than with the decreasing stress history. The large difference in these values is unexpected since it contradicts current theories of cumulative fatigue which ignore the effects of the order of stress history on the life of a component. 2. The method of equivalent reliabilities gives results which are consistant regardless of the order of application. There remains a need to experimentally verify the applicability of these methods.

3.3.4 Conclusions

Cumulative fatigue and its effect on component reliability are quite complicated and not completely understood areas. Several theories have been developed with most research devoted to understanding changes in material structure due to microscopic crack nucleation and propagation. Miner's rule [15] provides a rough approximation for determining the expected mean life of a component subjected to various stress levels. However, this rule does not provide a method for calculating component reliability.

The equation developed by Corten and Dolan [16] provides an improvement of Miner's rule in that it correlates better with existing data. This method suffers from lack of accuracy in providing a deterministic value from distributional data.

Work done under the NERVA program [17] to gain insight when stresses are applied in various sequences, has potential when combined with distributional fatigue life data.

The studies performed by Sorensen [18] and Serensen [19] approach the problem of fatigue using randomly applied or distributed loads resulting in distributed stress, which would constitute a generalized approach to cumulative fatigue. Their methods should be investigated further using our distributional fatigue data.

The first method proposed here for calculating cumulative fatigue reliability combines the product rule and conditional probability theory for sequentially applied levels of alternating stress. This method gives significantly different results for sequentially decreasing than increasing stress levels, namely 0.9_{21} and 0.9003 respectively. Thus this method does not appear to provide a general enough model.

The second proposed method is that of equivalent reliabilities where the equivalent number of cycles at each succeeding stress level is found for the reliability calculated at the preceding level. It includes consideration of fatigue damage at all stress levels and the effects of increasing versus decreasing stress history. The calculated reliabilities are essentially the same, thereby demonstrating consistency. This method is considered to provide a protentially valid general reliability model for cumulative fatigue.

4. OVERALL CONCLUSIONS

1. A methodology has been developed for designing specified reliabilities at optimum size and weight into rotating mechanical components subjected to fatigue under combined alternating bending stress and constant shear stress.
2. Three research machines have been designed and fabricated which are capable of simultaneously applying desired levels of alternating bending moment and constant torque to rotating test specimens.
3. Phase I of an experimental fatigue research program to verify the probabilistic design methodology was conducted with AISI 4340 steel specimens grooved to provide a theoretical stress concentration factor of 1.42. The data obtained were reduced using three computer programs developed for the CDC 6400 computer. The results were used to construct a distributional Goodman strength diagram for 2.5×10^6 cycles of life and distributional alternating bending stress versus cycles-to-failure (S-N) diagrams. The following specific conclusions were reached:
   3.1 The sample sizes of 12 and 18 were not sufficiently large for goodness-of-fit tests to determine whether the normal or the log-normal distribution provided a better fit to the data. The Chi-Squared test could not be used all the time and the Kolmogorov-Smirnov test did not reject either distribution. Phenomenological considerations, combined with goodness-of-fit test results, and coefficients of

skewness and kurtosis indicated that the log normal distribution provided the better fit to cycles-to-failure data, while the normal distribution provided the better fit to endurance strength and finite life stress-to-failure data.

3.2 *Probabilistic S-N diagrams plotted for stress ratios of ∞,* 3.5, 0.83 and 0.44 for Phase I data showed a linear relationship between the log of cycles-to-failure and the log of alternating bending stress levels for each stress ratio.

3.3 There is significant reduction in mean fatigue life as the alternating bending strength is reduced from ∞ to 0.44. For the alternating stress level of 70,000 psi the estimated cycles to failure, N_f, are 200,000 for an r_s of ∞, N_f = 84,000 for r_s of 3.5, N_f = 80,000 for r_s of 0.83, and N_f = 51,000 for r_s of 0.44.

3.4 There is a relatively high variability in the cycles-to-failure data with a coefficient of variation ranging from 15% to 25%. However, there is sufficent consistency to provide the essential linearity in the probabilistic envelope of the S-N curve, as well as in the plot of the mean log cycles.

3.5 The distributional Goodman strength diagram for Phase I results provides data directly useable by the designer for determinining the reliability of rotating components, for a service life of 2.5×10^6 cycles, subjected to combined

alternating bending stress and constant shear stress at various stress ratios for the given material and geometry of the specimen tested.

4. Phase II of the experimental fatigue research program was initiated and completed for specimens of identical specifications to the Phase I specimens, except for a different groove radius to provide a theoretical stress concentration factor of 2.34. Fatigue tests were accomplished for plotting S-N diagrams for the stress ratios of ∞, 1.06, 0.40, 0.25 and 0.15.
5. The computer programs were revised to incorporate the current calibration coefficients for the fatigue research machines, the application of a revised Chi-Squared goodness-of-fit test, and a subroutine for the plotting of a histogram and the estimated normal, lognormal, and Weibull distributions using a Cal-Comp plotter.
6. The conclusions reached from the results of the computer programs are the following:

   6.1 Increasing the sample size to 35 allowed the use of the Chi-Squared goodness-of-fit test which proved to be more discriminating than the Kolmogorov-Smirnov test. The Chi-Squared test rejected the normal distribution three times out of sixteen samples at the 0.05 level of significance; whereas the lognormal distribution was rejected only two times out of sixteen. Thus, the previous tenuous preference for using the lognormal distribution for cycles-to-failure data was strengthened. For the Chi-Squared test

to be valid, it must have five or more observations in each cell; this requires that the end cells sometimes be combined. The existing methodology describes the tails of distributions with about 5% accuracy. A method to improve the accuracy of describing the tails of distributions for use in calculating reliability is needed.

6.2 Comparison of the S-N and Goodman diagrams for Phase I and Phase II research reveals that the endurance strength for 2.5×10^6 cycles of life and the finite fatigue life at a specified alternating stress level are significantly reduced when the stress concentration is increased by reducing the groove radius. The S-N diagrams show that the fatigue life for an alternating stress level of 80,000 psi decreases from 75,000 cycles for Phase 1 specimens to 9,000 cycles for Phase II specimens for a stress ratio of ∞. In Phase I tests, an alternating stress level of 50,000 psi at the stress ratio of 0.44 approached the endurance level, and a mean cycles to failure could not be determined. For the Phase II tests at the stress ratio of 0.40, the 50,000 psi alternating stress level approached the highest stress level that could be tested without suffering shear failures during machine set up, and at this stress level the mean cycles-to-failure was about 56,000 cycles.

6.3 The cycles-to-failure results emphasize the effect of stress ratio on the fatigue life of specimens at specified levels of alternating stress and the maximum stress levels at

which fatigue tests can be conducted. The upper limits for the tests ranged from about 120,000 psi for alternating bending stress for the stress ratio of ∞ to approximately 32,000 psi for the stress ratio of 0.15. Examination of the S-N diagrams show that there is a limited useable range for low stress ratios; consequently, the stress level and stress ratio become very critical in a component design using low stress ratios.

7. Generation and analysis of finite life distributional Goodman diagrams reveal that the conventional deterministic Goodman diagram is extremently conservative.

8. The FORTRAN Computer program to estimate the parameters of the three-parameter Weibull distribution which would represent cycles-to-failure data is providing good results. The program incorporates the K-S and Chi-Squared goodness-of-fit tests. The K-S test did not reject the Weibull in 17 tests, and the Chi- Squared test rejected the Weibull 5 times in 17 tests. Based on these test results, the Weibull can be considered a useable distribution for cycles-to-failure distributions, although the lognormal is favored by the Chi-Squared test.

9. Cumulative fatigue is a complicated and not completely understood design area. Several theories have been developed with most research devoted to understanding changes in material structure due to microscopic crack nucleation and progression. Two methods were investigated in this research for calculating the reliability of a component sequentially subjected to a number of stress levels.

The method of equivalent reliabilities provides consistent component reliabilities regardless of the order of application of the different stress levels.

5. RECOMMENDATIONS

1. The Phase II type experimental research program should be repeated for other geometries and materials, and a complete set of S-N and Goodman diagrams should be prepared.
2. An analytical study should be made in search of a mathematical relationship between infinite life and finite life Goodman diagrams.
3. An analytical study should be made in search of a mathematical model for the effect of varying stress concentrations and stress ratios in fatigue tests.
4. Further study should be made of infinite and finite life Goodman strength diagrams to determine (1) a more accurate model than the ellipse and (2) if another failure theory, or combination of theories, is more valid for fatigue failure than the distortion energy theory.
5. A study should be made with sample sizes larger than 35 to determine the effect of sample size on describing the tails of the statistical distributions. As part of the study an error analysis should be made comparing the predicted reliabilities using the lognormal and Weibull distributions for the cycles to failure data and for different sample sizes.
6. A Phase III experimental research program should be conducted using ungrooved specimens so that the true effect of stress concentration on fatigue life may be better quantified comparatively.

7. An experimental cumulative fatigue research program should be conducted by sequentially applying various levels of stresses for specified numbers of cycles and to specimens with different groove radii. The data thus obtained should be used to determine the validity of the equivalent reliabilities cumulative fatigue model.

ACKNOWLEDGEMENTS

The contributions of the following persons to the efforts of this final report are gratefully acknowledged:

1. Mr. Vincent R. Lalli, NASA Project Manager
2. Messrs. Jim Blair, Michael Carrell, Scott Clemans, Steve Cressey, Thomas Dodge, Everett Gardner, William Guerrieri, John Kempert, Robert McLaughlin, Roger Neill, Wesley Shonerd, Anu Vaze, Robert Warner and Frank Watts all from The University of Arizona.

REFERENCES

1. Mabie, H. H., and Gjesdahl, M. S.; "A Fatigue Testing Machine for Reversed Bending and Steady Torque," Proc. SESA, XIV(1), pp. 83-88, 1956.

2. Kececioglu, Dimitri B., Dr.; et al, "A Probabilistic Method of Designing Specified Reliabilities into Mechanical Components with Time Dependent Stress and Strength Distributions," The University of Arizona, NASA CR-72836, 1967, 331 pp.

3. Kececioglu, Dimitri B., et al.; "Design and Development of and Results from Combined Bending Torsion Fatigue Reliability Research Machines," The University of Arizona, NASA CR-72838, 1969, 57 pp.

4. Kececioglu, Dimitri B., Dr.; and McConnell, J. B., "Calibration of Combined Bending Torsion Fatigue Reliability Research Machines and Reliability Data Reduction," The University of Arizona, NASA CR-72839, 1969, 151 pp.

5. Kececioglu, Dimitri B., and Smith, J. L.; "Statistical Complex Fatigue Data for AISI 4340 Steel and its Use in Design by Reliability," The University of Arizona, NASA CR-72835, 1970, 175 pp.

6. Mood, A. M., and Dixon, W. K.; "A Method for Obtaining and Analyzing Sensitivity Data," Journal of the American Statistical Association, Vol. 43, 1948, pp. 109-126.

7. Spiegel, Murray R.; <u>Theory and Problems of Statistics</u>, Schaum Publishing Co., 1961, 359 pp.

8. Fryer, H. C.; Concepts and Methods of Experimental Statistics, Allyn and Bacon, Inc., Boston 1966, 602 pp.

9. Shigley, J. E.; Mechanical Engineering Design, McGraw-Hill Book Comapny, New York, N.Y., 1963, 631 pp.

10. Kececioglu, D. B., Dr. and Guerrieri, W. W.; "Generation of Finite Life Distributional Goodman Diagrams for Reliability Prediction," The University of Arizona, NASA CR-120813, 1971, 240 pp.

11. Hahn, G. J.; and Shapiro, S. S.; Statistical Models in Engineering, John Wiley and Sons, New York, N.Y., 1967, 355 pp.

12. Lochner, Robert H.; "When and How to Use The Weibull Distribution," Ninth Annual Reliability Engineering and Management Institute Lecture Notes, The University of Arizona, November 1971, 45 pp.

13. Sturges, H. A., "The Choice of a Class Interval," Jl. American Statistical Association, Vol. 21, 1926, pp. 65-66.

14. Osgood, Carl C.; Fatigue Design, John Wiley and Sons, Inc., New York, N.Y., 1970, 523 pp.

15. Miner, A. M.; "Cumulative Damage in Fatigue," Trans. ASME, Vol. 67, A 159, 1945, pp. 343-355.

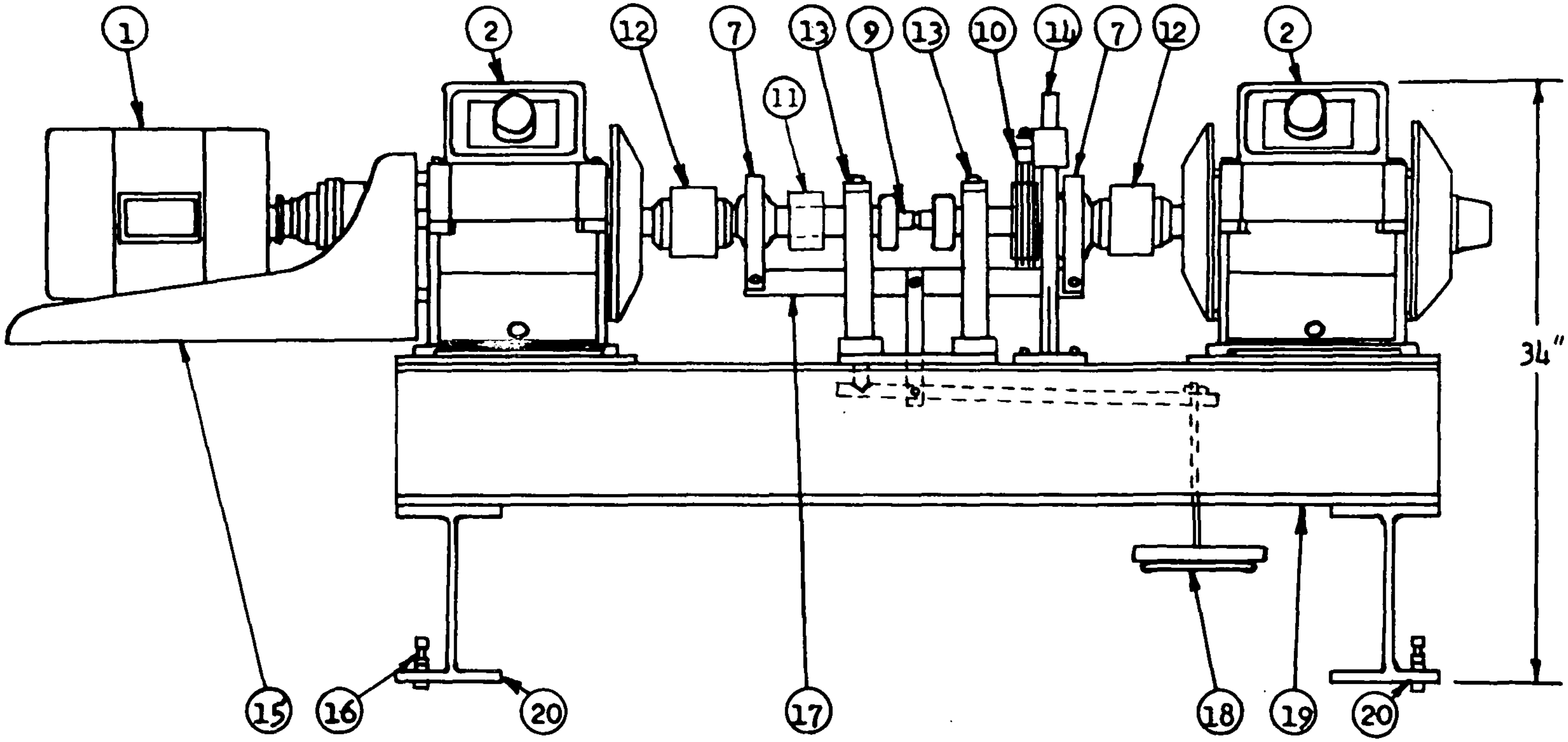
16. Corten, H. T., and Dolan, T. J.; "Cumulative Fatigue Damage," International Conference of Fatigue of Metals, The Institute of Mechanical Engineers and American Society of Mechanical Engineers, London, England, November 1956, 14 pp.

17. "Simplified PRE-PDR Techniques for Assessing Components Reliability," NERVA, Aerojet Nuclear System Company, Reliability Calculations for the Cases of Combined Stress and Fatigue Loading, 1970, 62 pp.

18. Sorensen, Arthur, Jr., "A General Theory of Fatigue Damage Accumulation," Transactions ASME Winter Annual Meeting, New York, December 1-5, 1968, pp. 420-434.

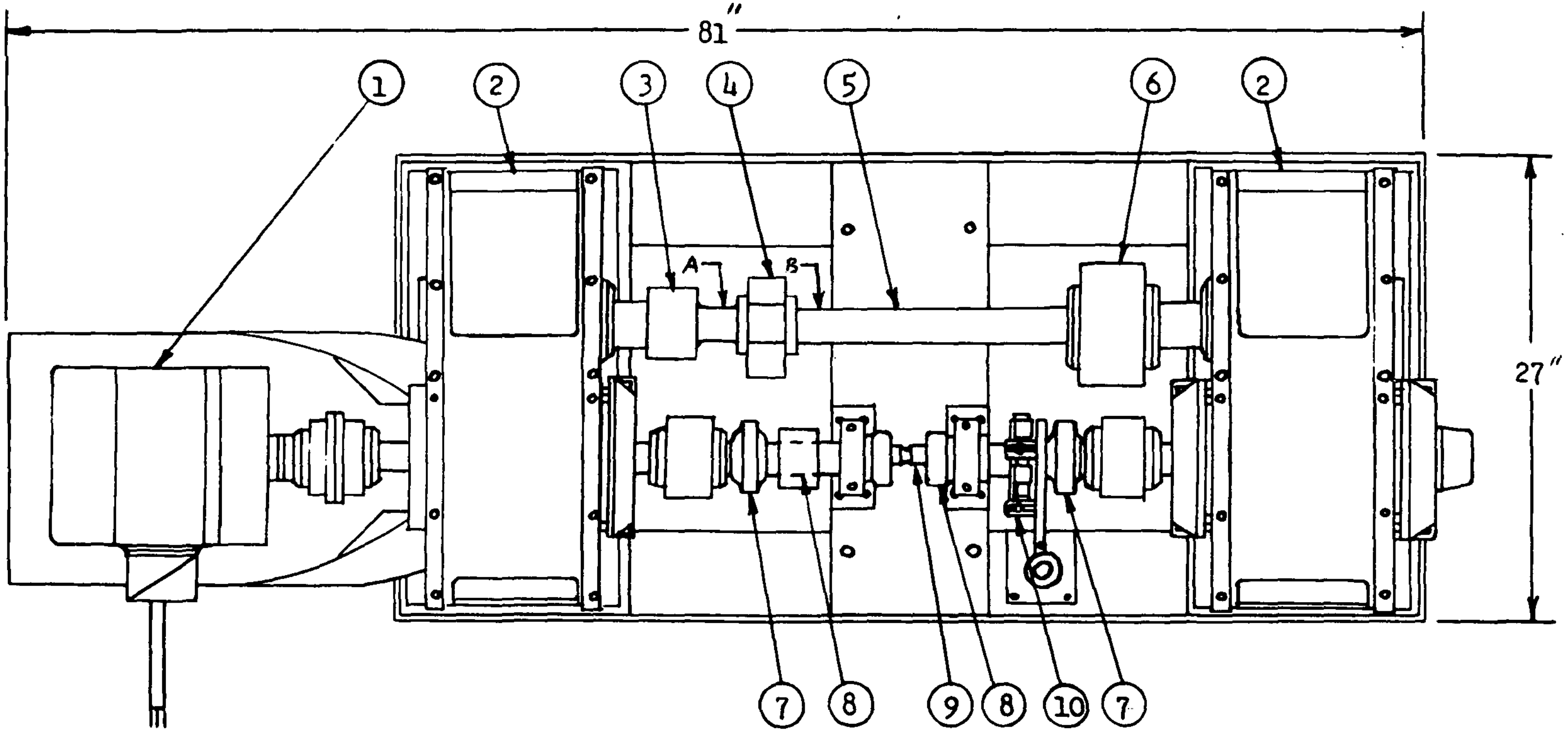
19. Serensen, S. V.; "Fatigue Damage Accumulation and Safety Factors Under Random Variable Loading," presented at the International Conference held in Warsaw, and published in the Conference Proceedings, pp. 33-42, published under the conference title "Fatigue Resistance of Materials and Metal Structural Parts," by The Macmillan Co., New York, 1964, 313 pp., Alfred Buch, Editor.

FIGURES



1. Motor
2. Gearboxes
7. Loading Frame Bearing
9. Test Specimen
10. Slip Rings and Brushes
11. Slipring Counterweight
12. Coupling
13. Safety Bridge
14. Brush Holder
15. Motor Bracket
16. Leveling Bolts
17. Loading Frame
18. Pan and Weights for Bending Load
19. Machine Mount
20. Floor Mounts

Fig. 1 Schematic front view of combined reversed bending-steady torque fatigue-reliability research machine.



1. Motor
2. Gearboxes
3. Coupling
4. Infinit-Indexer for Torque Load
5. Backshaft
6. Coupling
7. Loading Frame Bearing
8. Straight Shank Toolholder
9. Test Specimen
10. Slip Rings and Brushes

Fig. 2 Schematic top view of combined reversed bending-steady torque fatigue-reliability research machine.

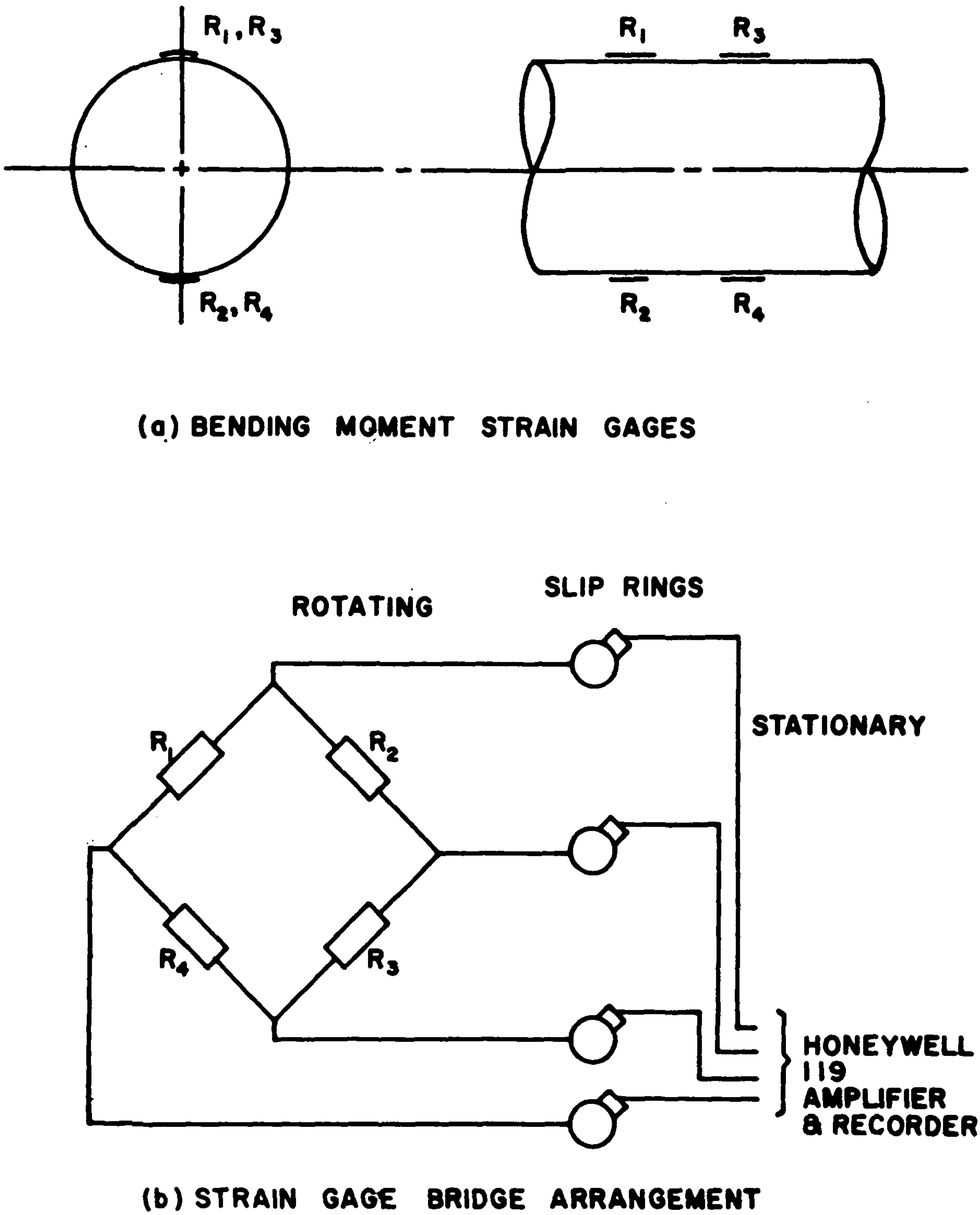
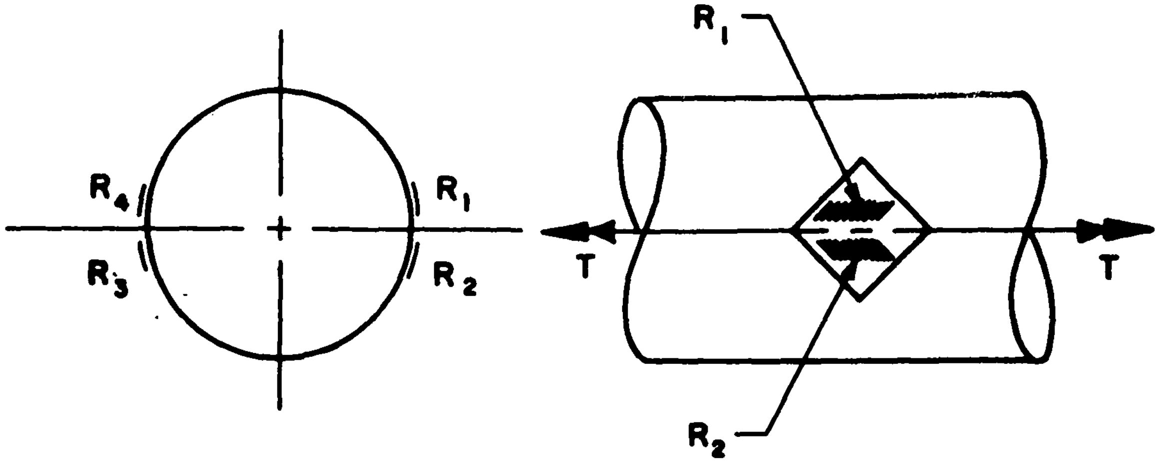
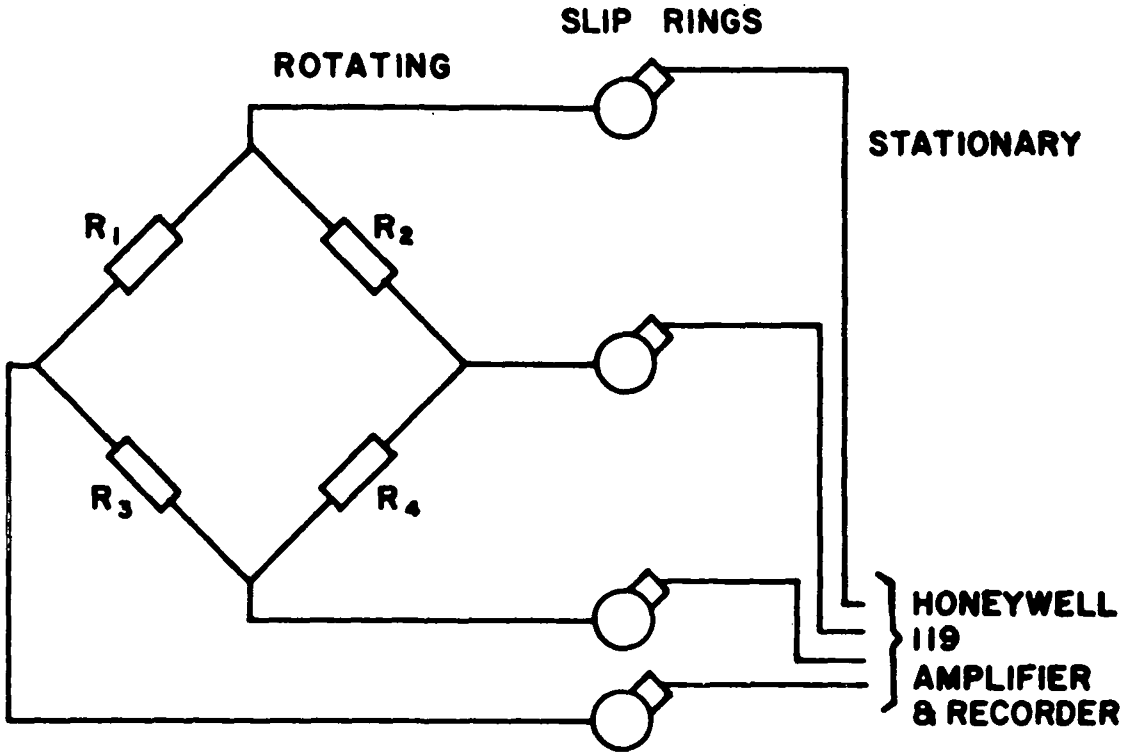


Fig. 3 Bending stress strain gage bridge.



(a) TORSION STRAIN GAGE



(b) STRAIN GAGE BRIDGE ARRANGEMENT

Fig. 4 Shear stress strain gage bridge.

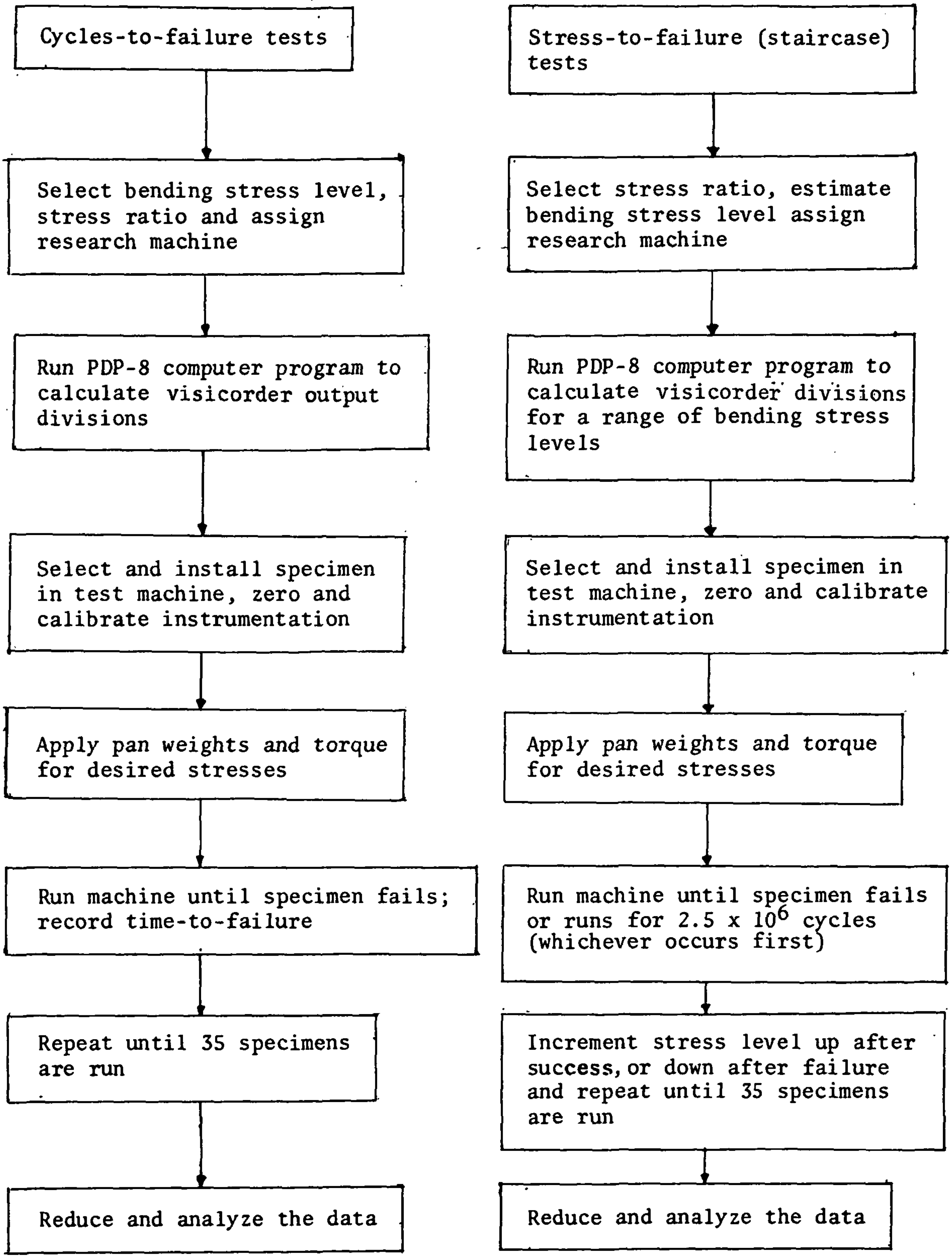


Fig. 5 Flow chart of steps in cycles-to-failure and stress-to-failure fatigue tests.

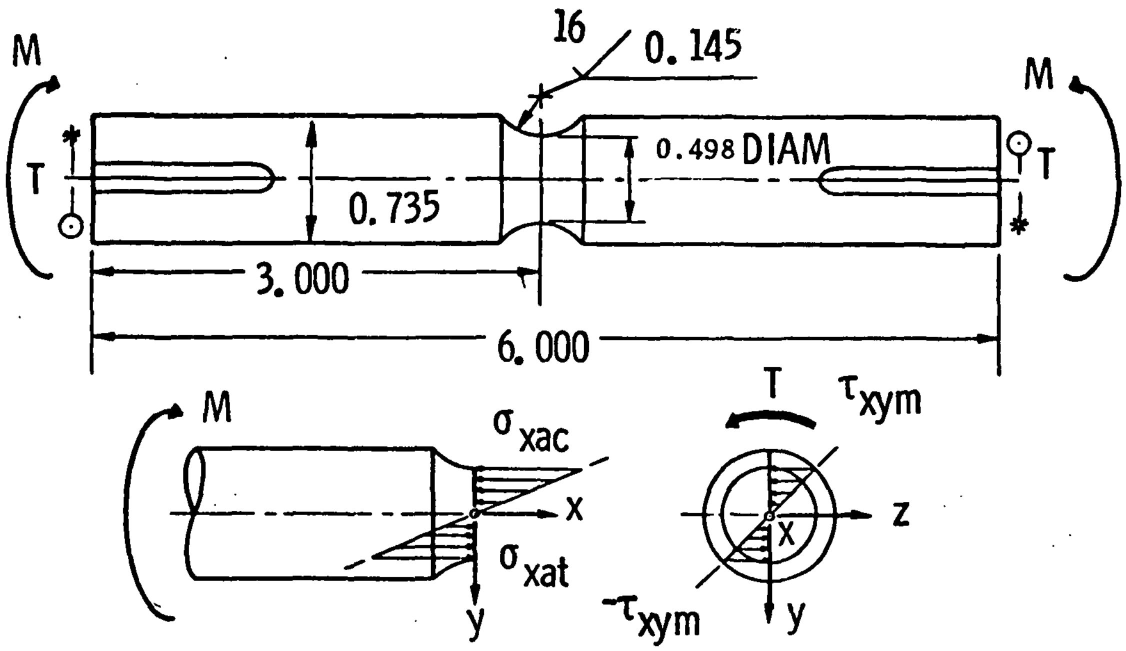


Fig. 6 Phase I research specimen geometry.

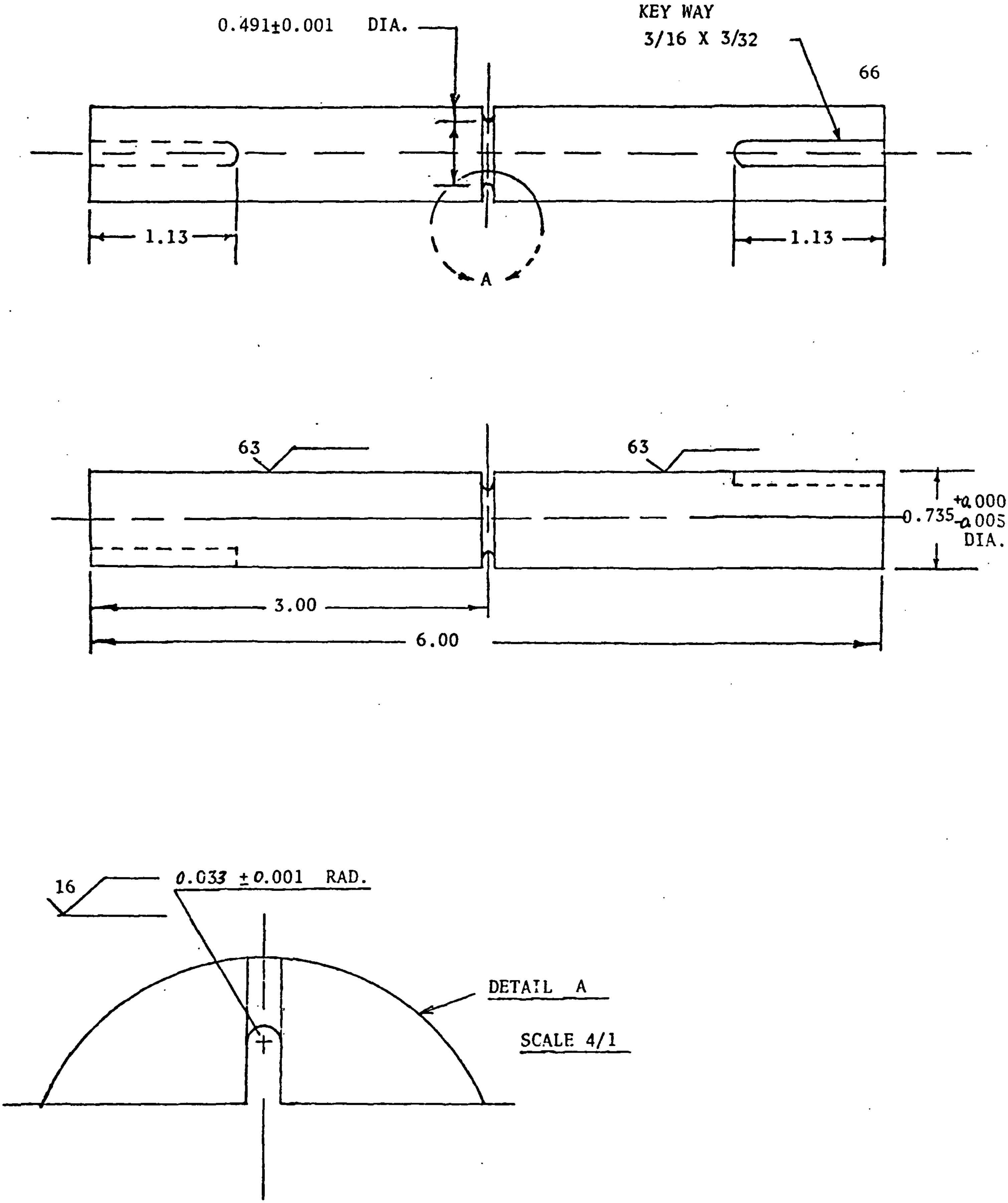


Fig. 7 Phase II research specimen geometry.

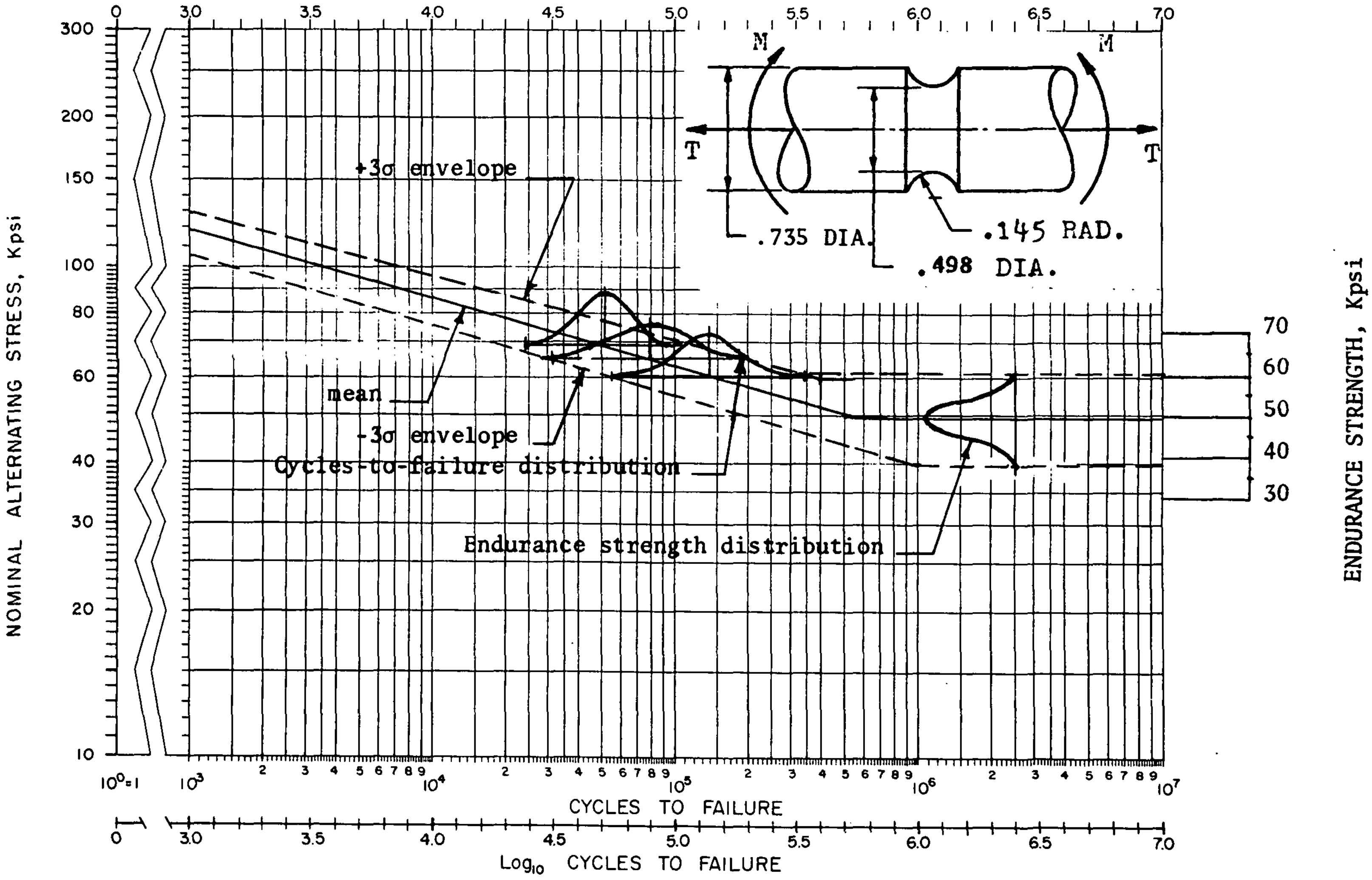


Fig. 8 Cycles-to-failure distributions at the stress ratio of 0.44 for AISI 4340 steel R_c 35/40 Phase I grooved specimens.

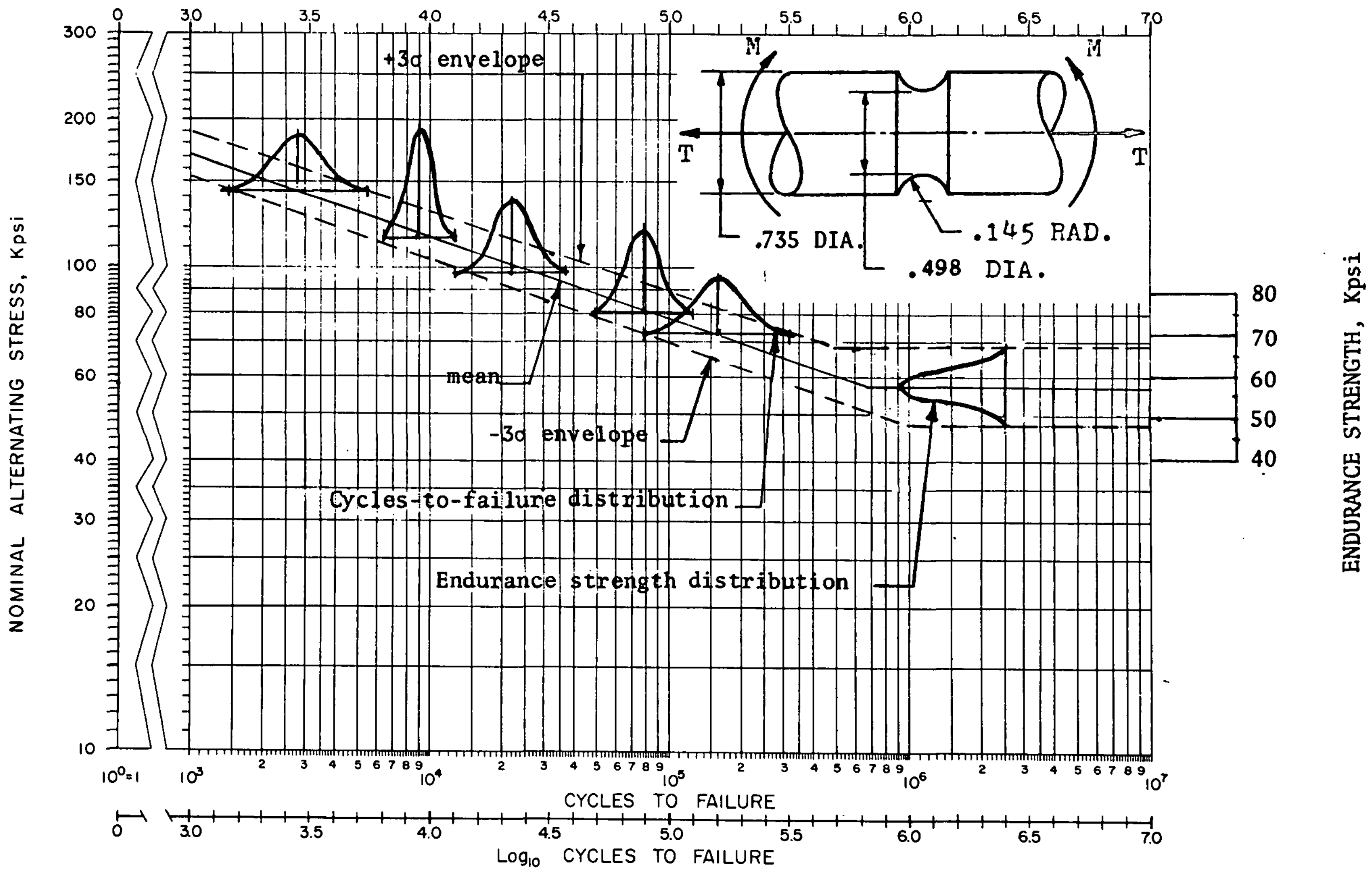


Fig. 9 Cycles-to-failure distributions at the stress ratio of ∞ for AISI 4340 steel R_C 35/40 Phase I grooved specimens.

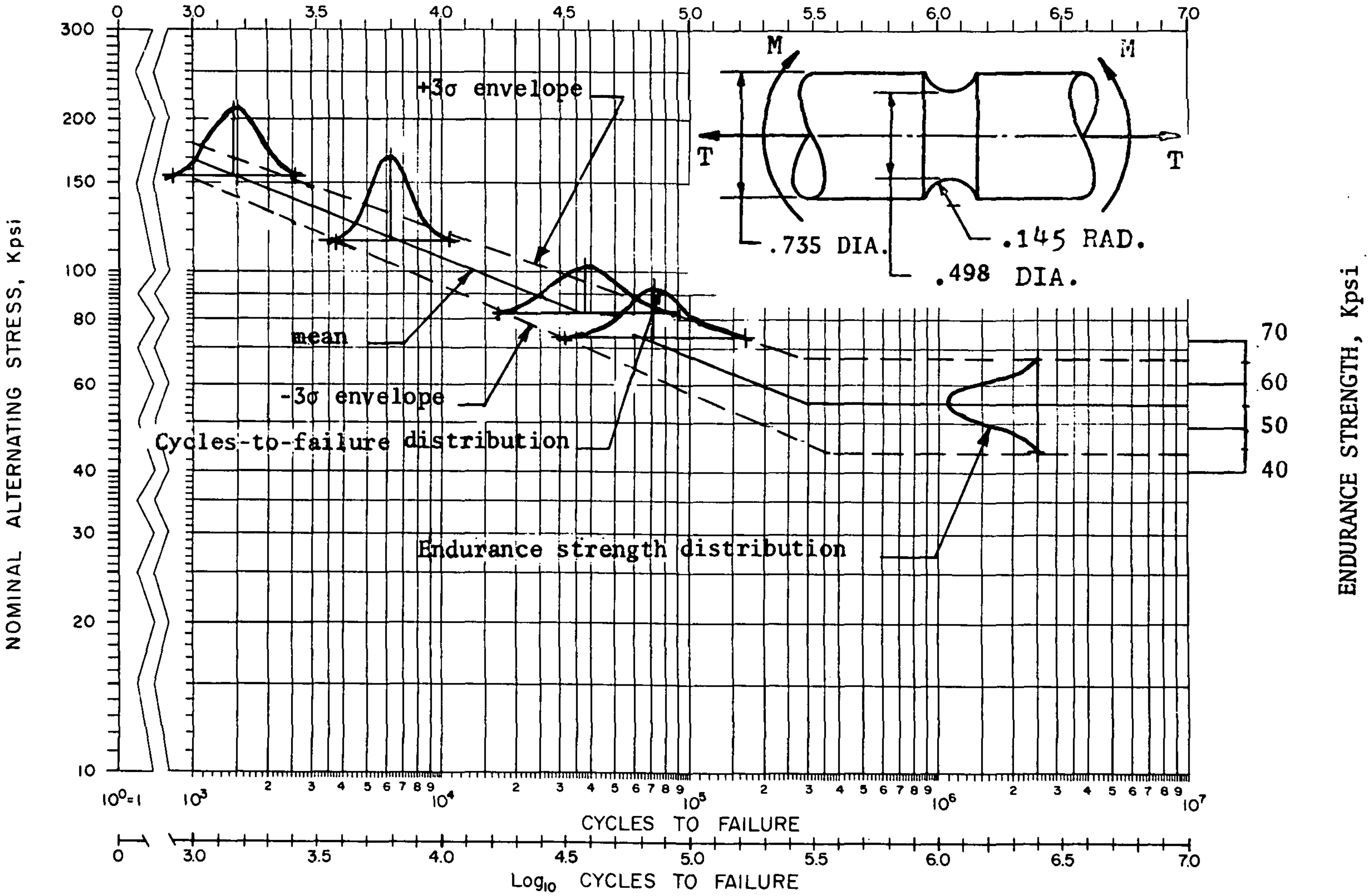


Fig. 10 Cycles-to-failure distributions at the stress ratio of 3.5 for AISI 4340 steel R_c 35/40 Phase I grooved specimens.

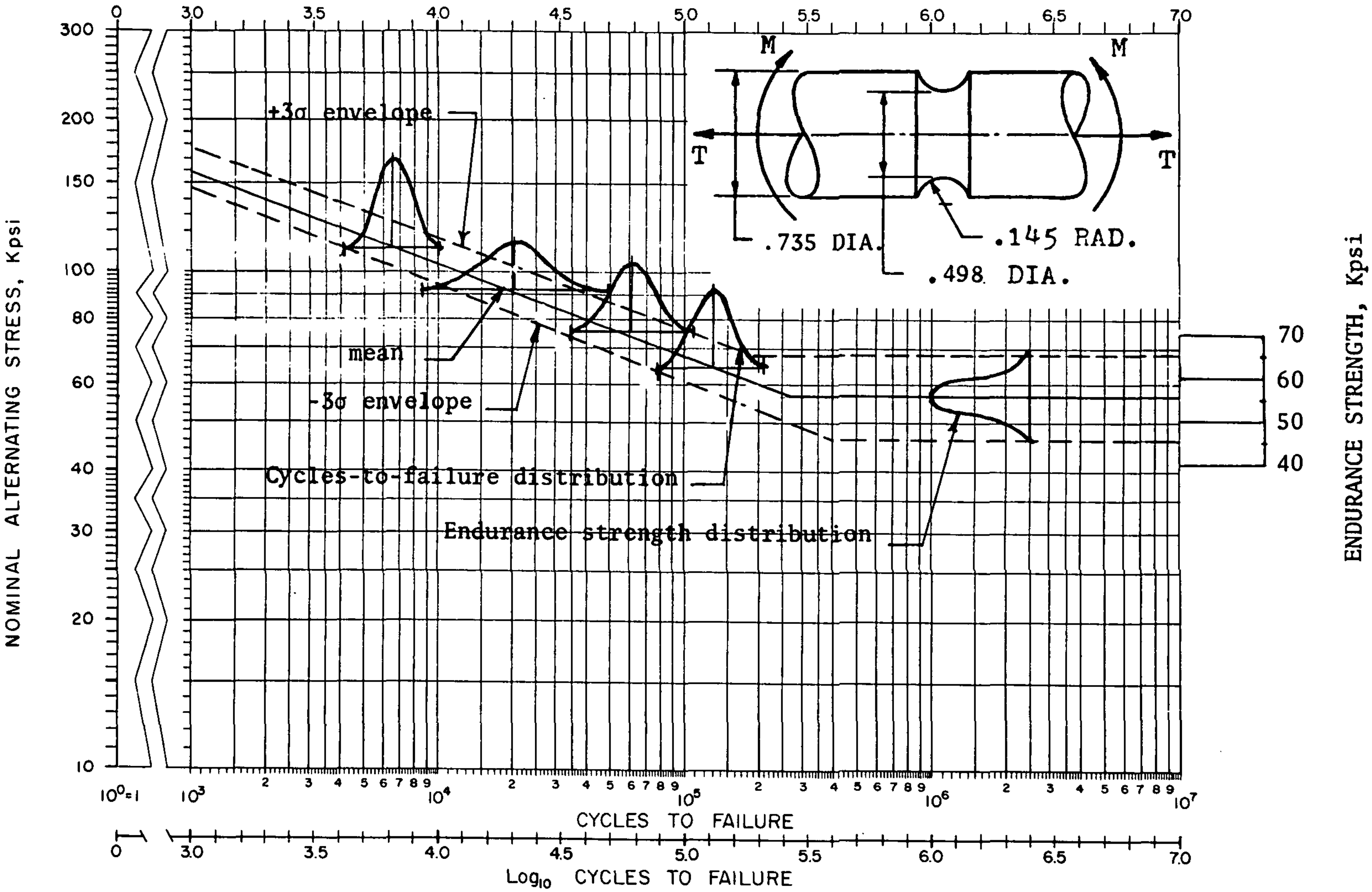


Fig. 11 Cycles-to-failure distributions at the stress ratio of 0.83 for AISI 4340 steel R_c 35/40 Phase I grooved specimens.

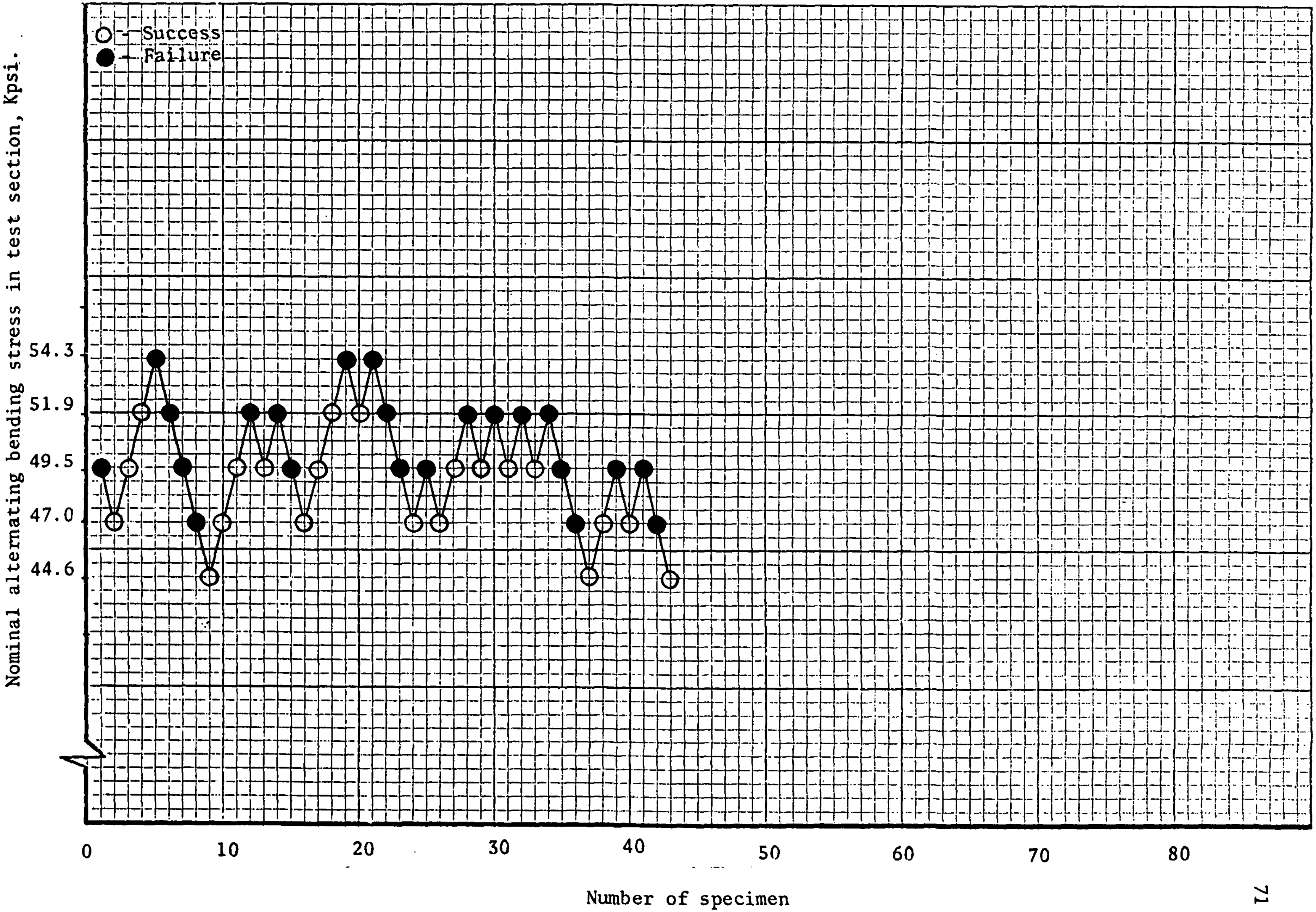


Fig. 12 Endurance strength data obtained by the staircase method for stress ratio of 0.45 for AISI 4340 steel, MIL-S-5000B, Condition C4, Rockwell C 35/40, with Phase I grooved specimens.

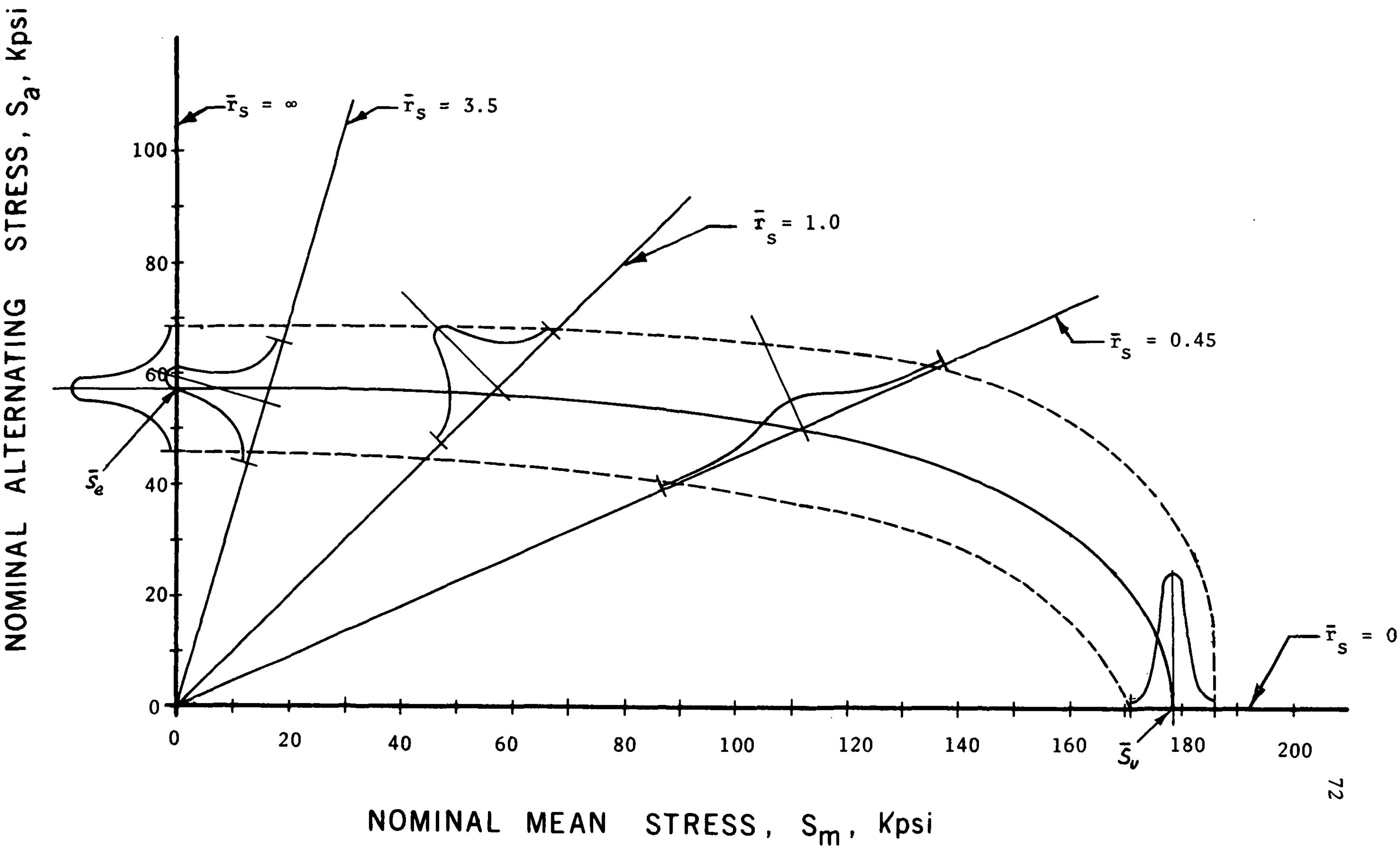


Fig. 13 Distributional Goodman strength diagram for 2.5 x 10^6 cycles of life Phase I results with AISI 4340 steel R_c 35/40 grooved specimens (See Table 10).

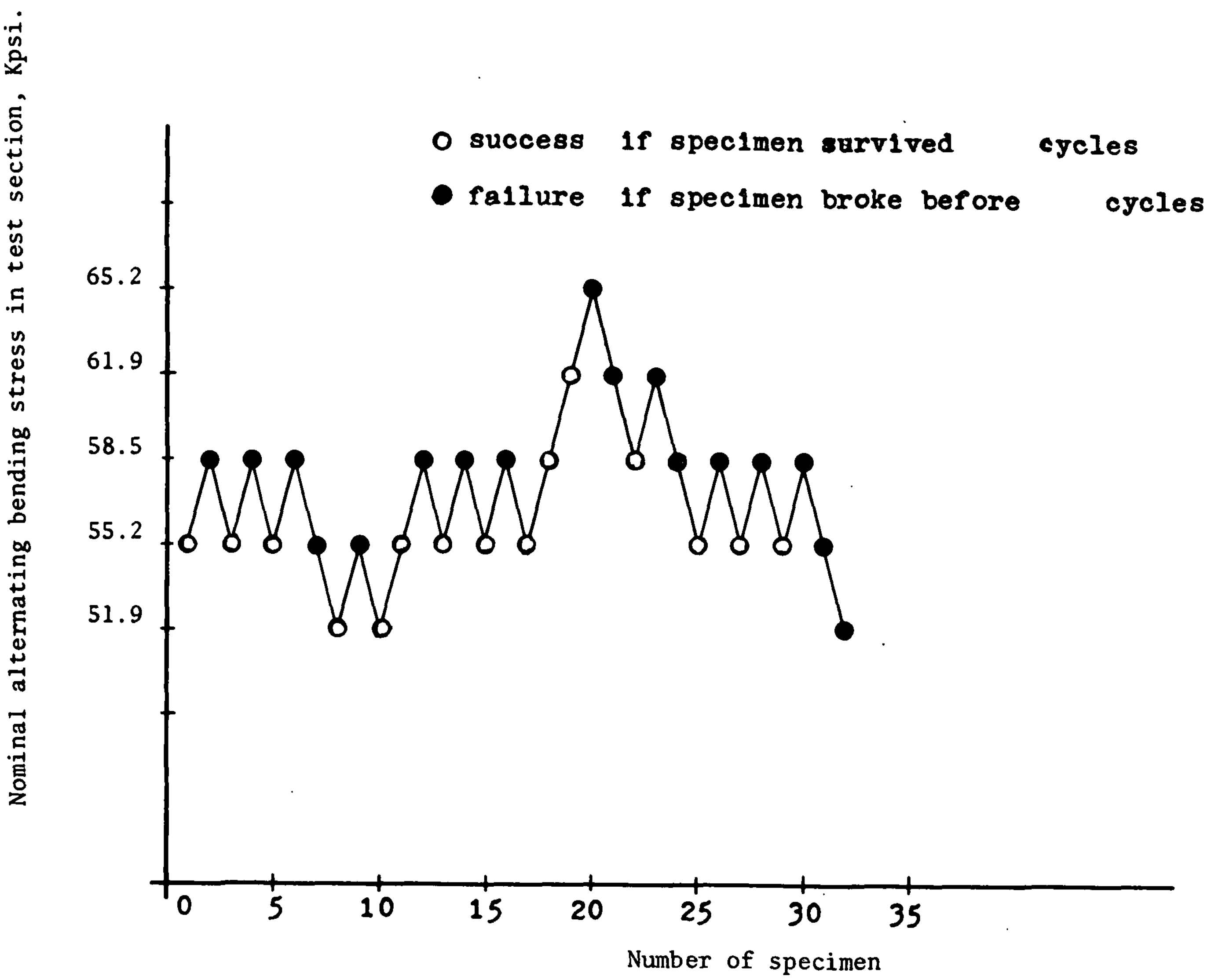


Fig. 14 Endurance strength data obtained by the staircase method for stress ratio of ∞ for AISI 4340 steel, MIL-S-5000B, Condition C4, Rockwell C 35/40, with Phase I grooved specimens.

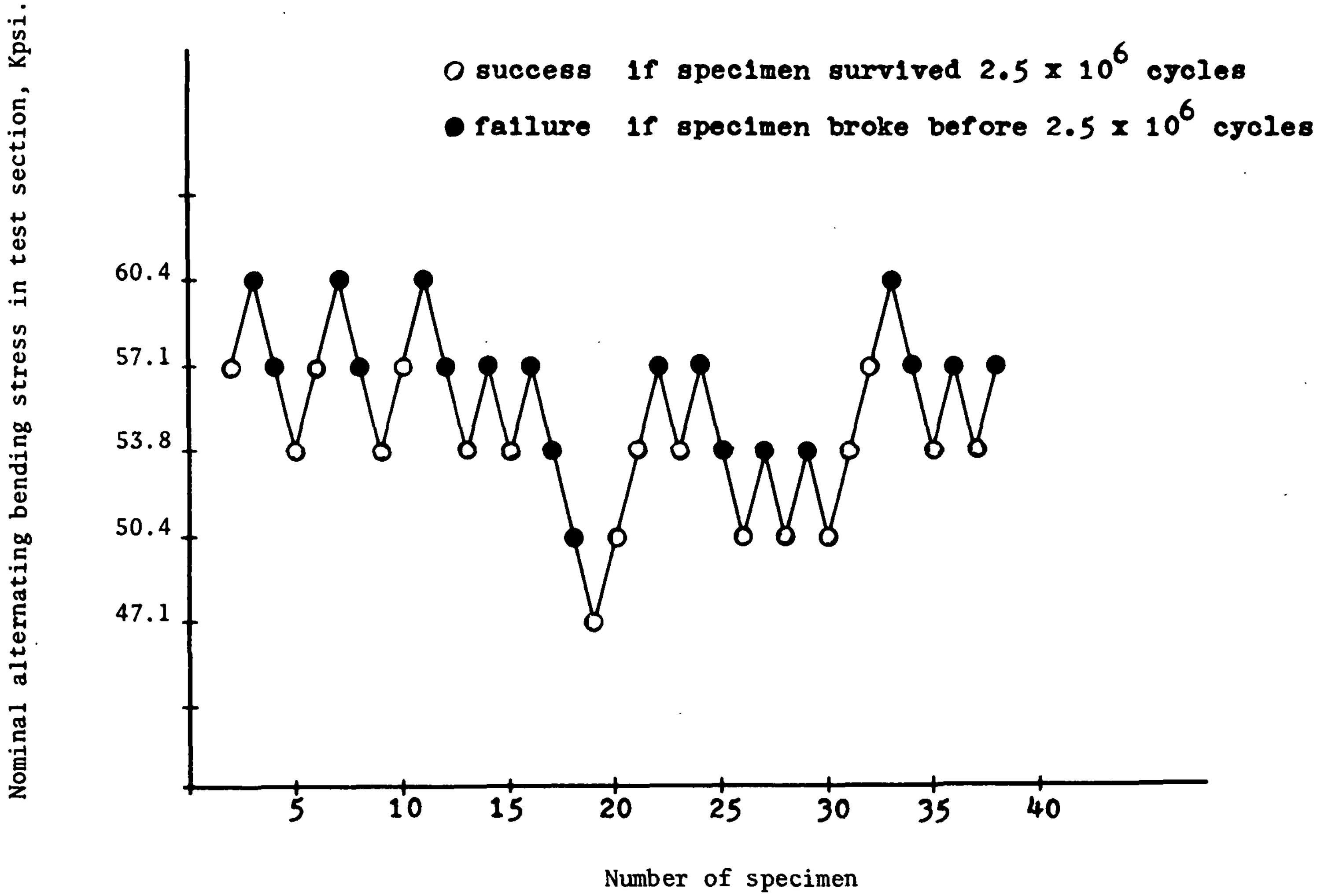


Fig. 15 Endurance strength data obtained by the staircase method for stress ratio of 3.5 for AISI 4340 steel, MIL-S-5000B, Condition C4, Rockwell C 35/40, with Phase I grooved specimens.

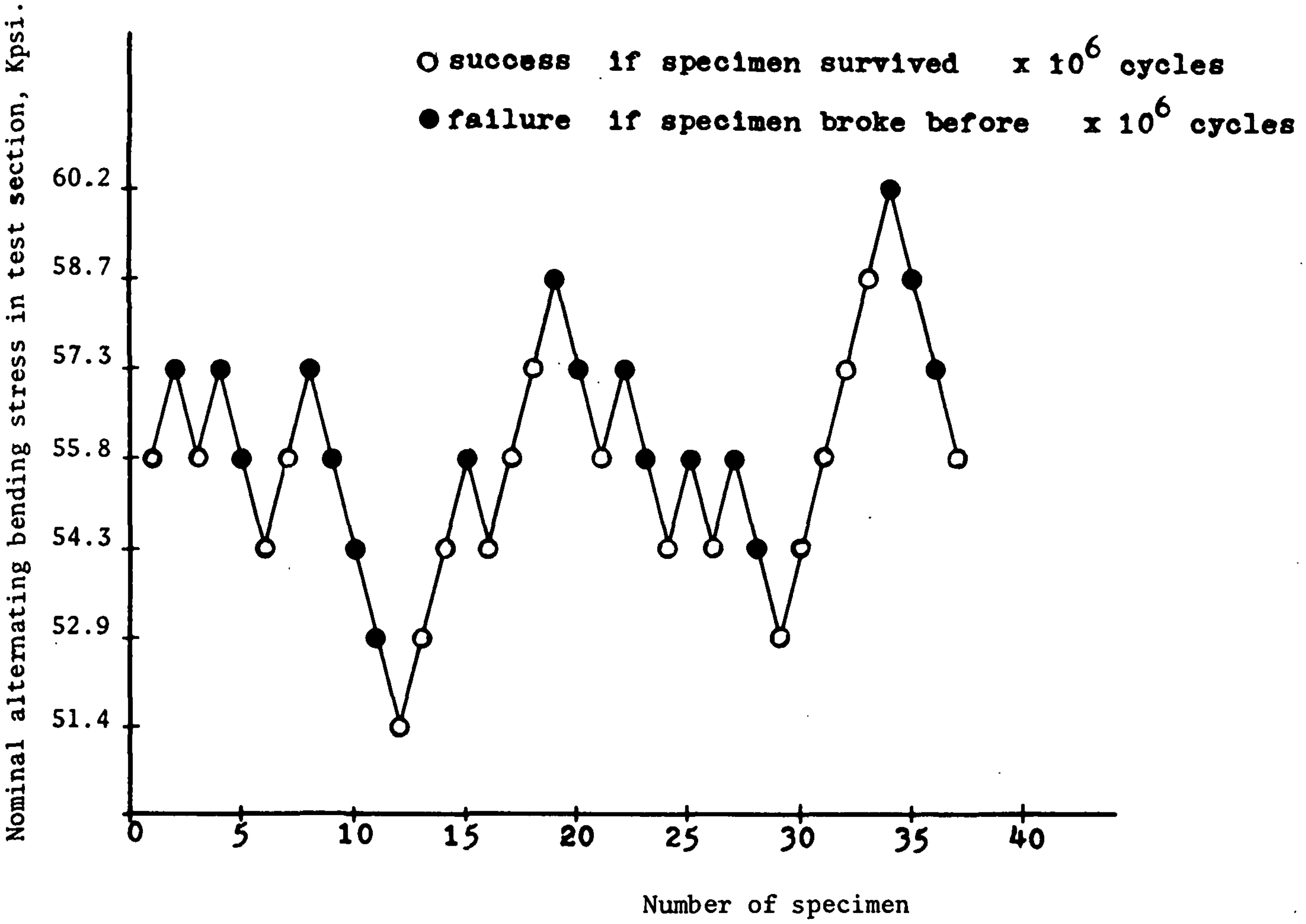
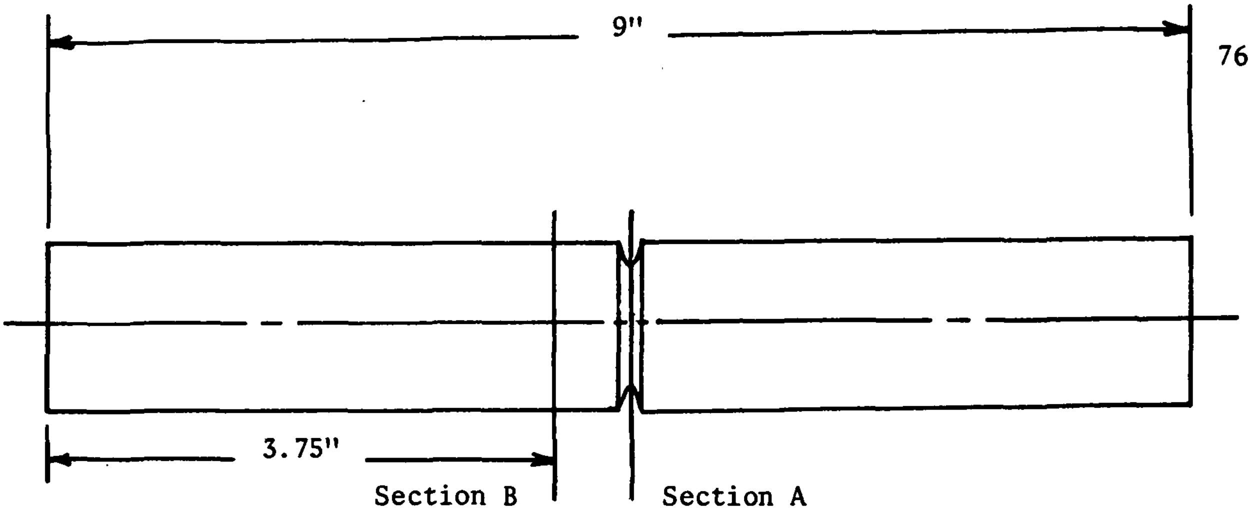
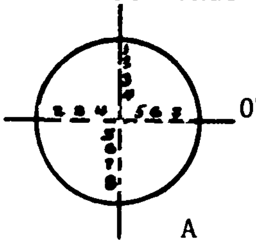


Fig. 16 Endurance strength data obtained by the staircase method for stress ratio of 1.0 for AISI 4340 steel, MIL-S-5000B,Condition C4, Rockwell C 35/40, with Phase I grooved specimens.

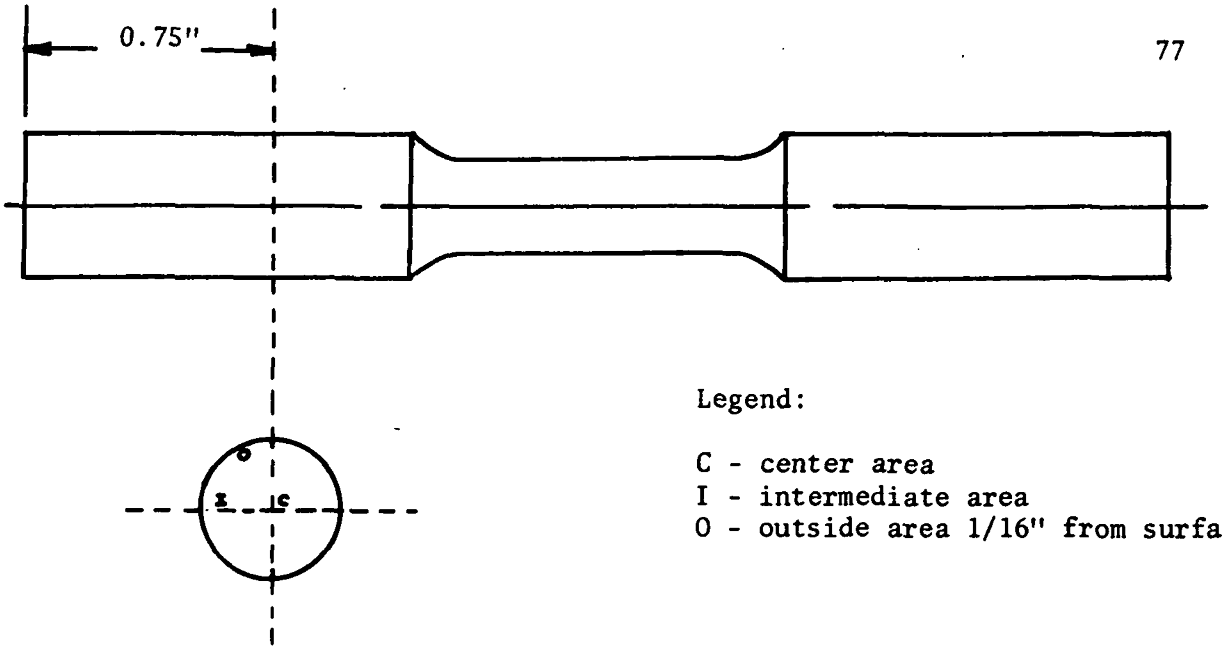


(a) The length AB used for hardness measurements.



(b) Test pattern for section A

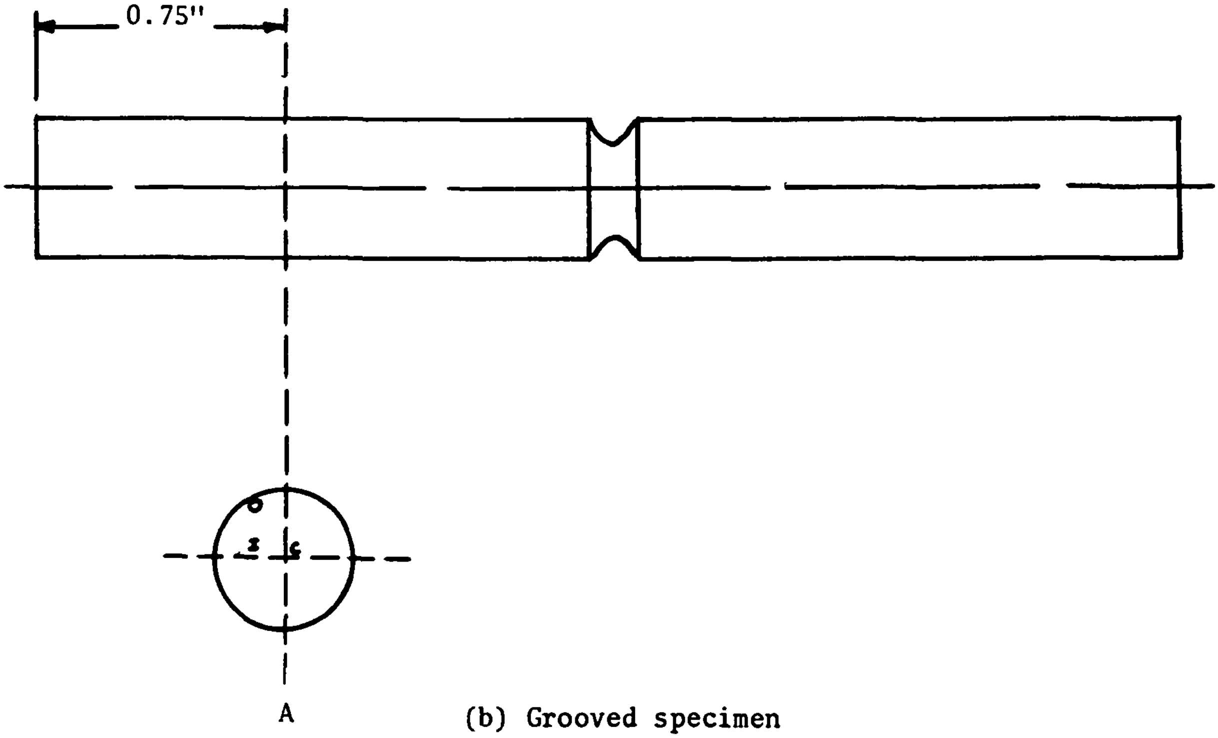
Fig. 17 Grooved specimen sections used for hardness measurements.



Legend:

C - center area
I - intermediate area
O - outside area 1/16" from surface

(a) Ungrooved specimen



(b) Grooved specimen

Fig. 18 Specimen sections and locations for hardness measurements.

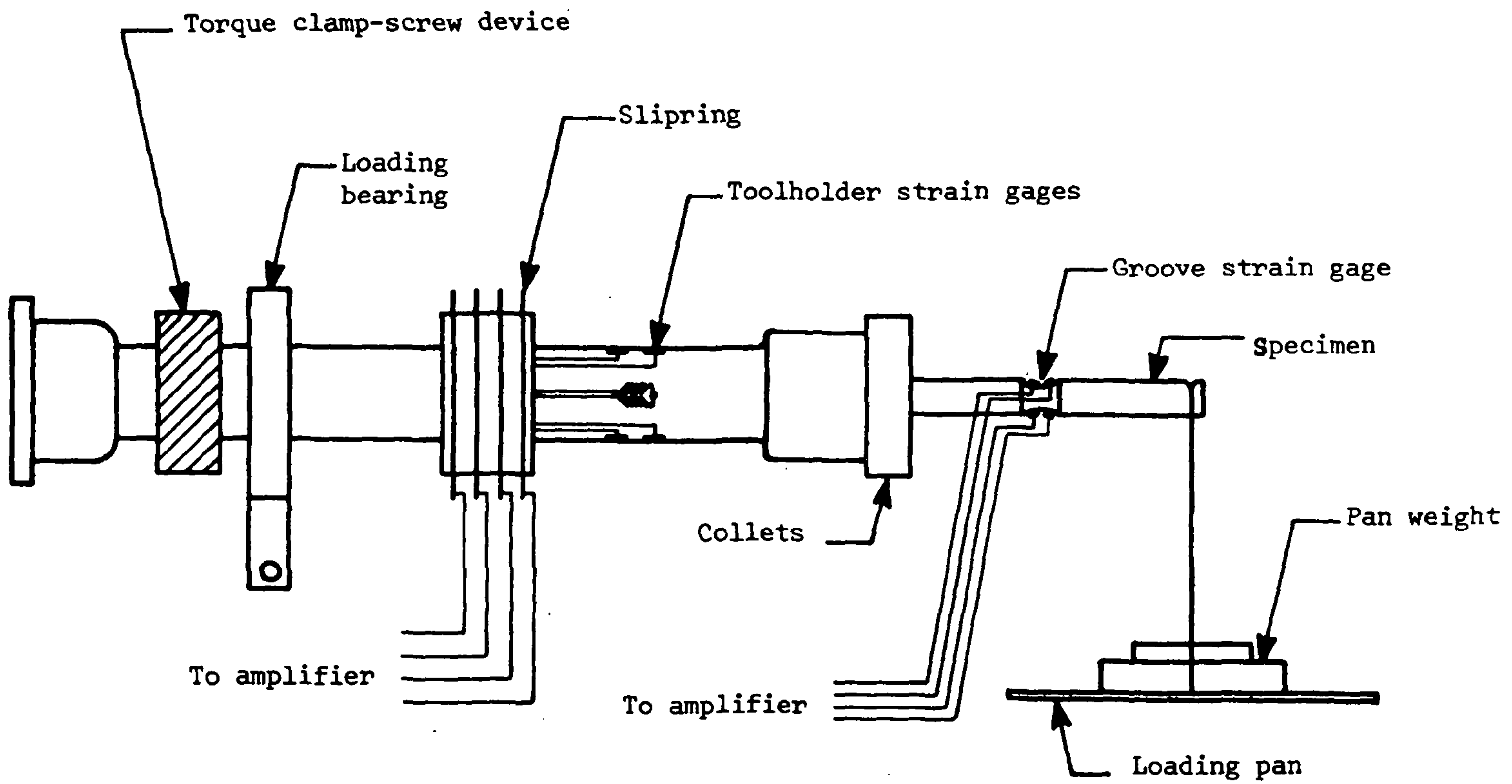


Fig. 19 Bending stress bridge calibration setup.

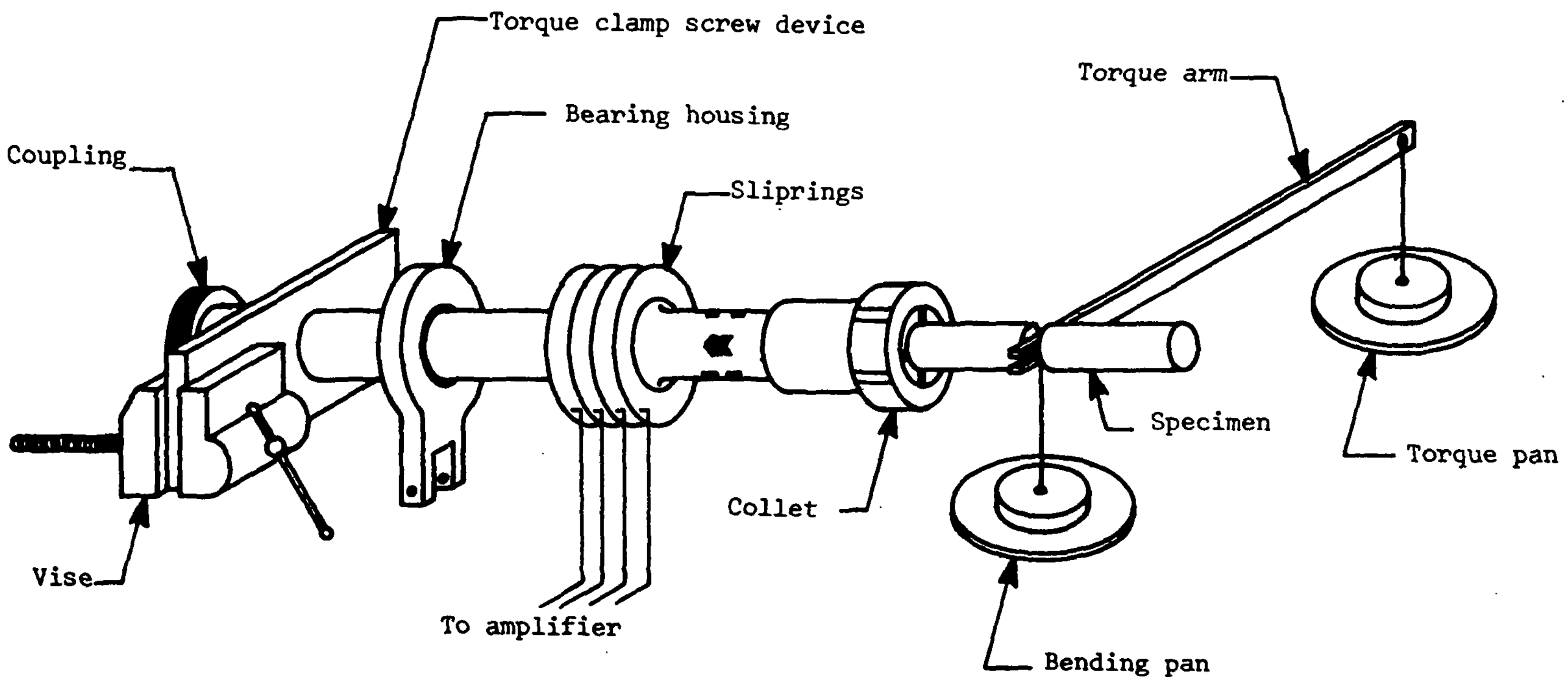


Fig. 20 Torque (shear) stress bridge calibration setup.

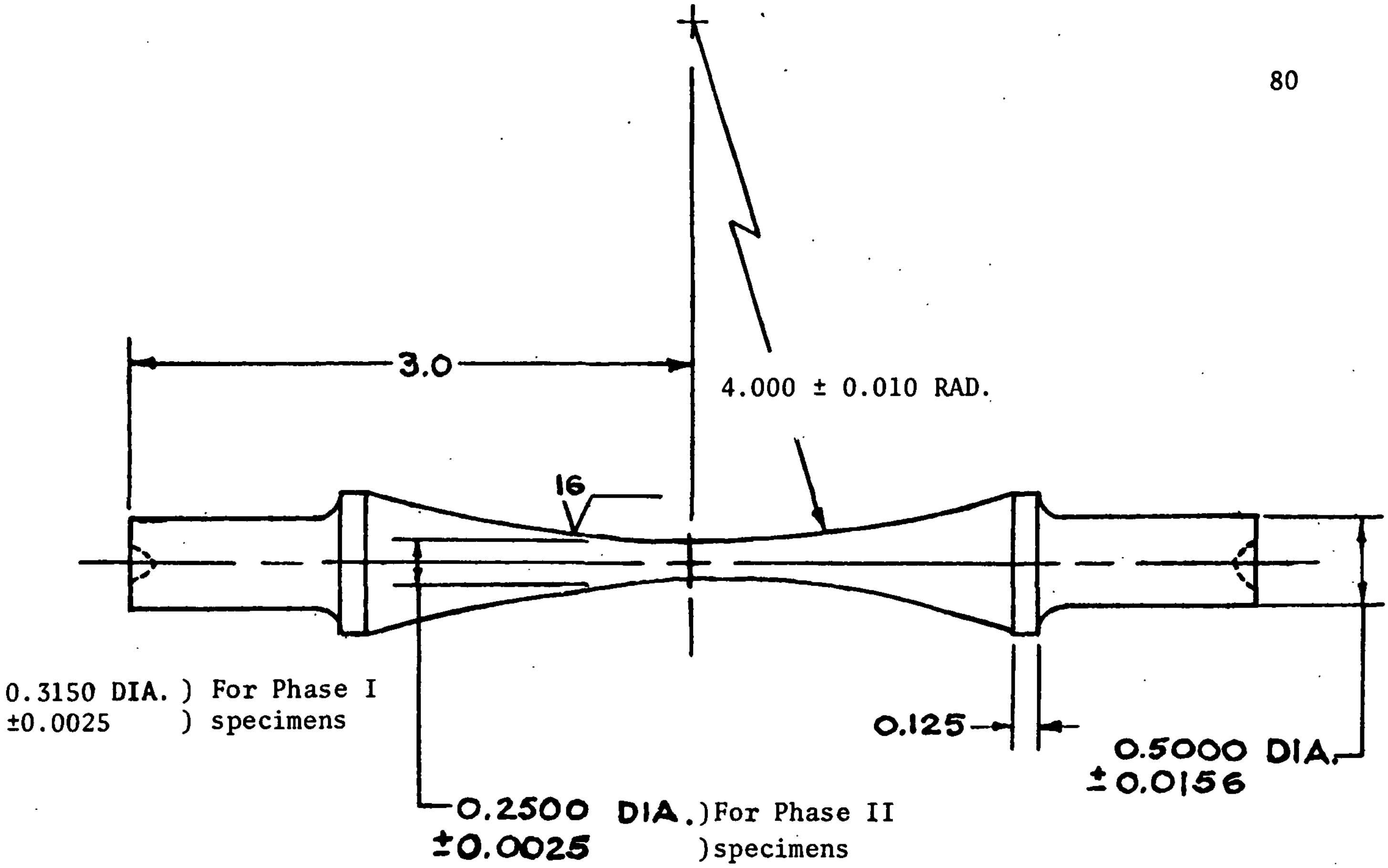


Fig. 21 Ungrooved endurance strength specimen.

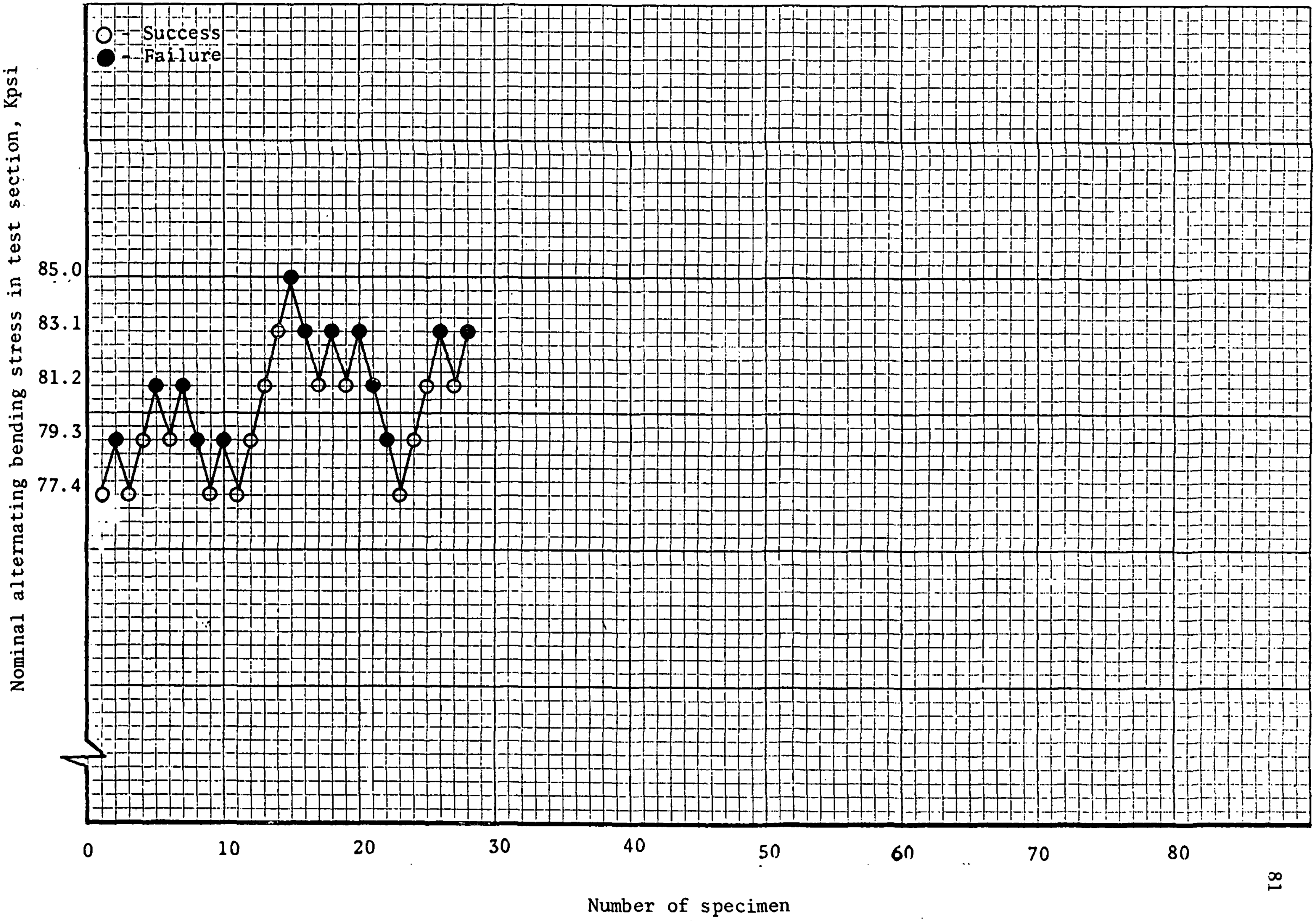


Fig. 22 Endurance strength data obtained by staircase method in Ann Arbor machine for the stress ratio of ∞ for AISI 4340 steel R_c 35/40 Phase I research. Specimens with no groove and a test section diameter of 0.3150 in.

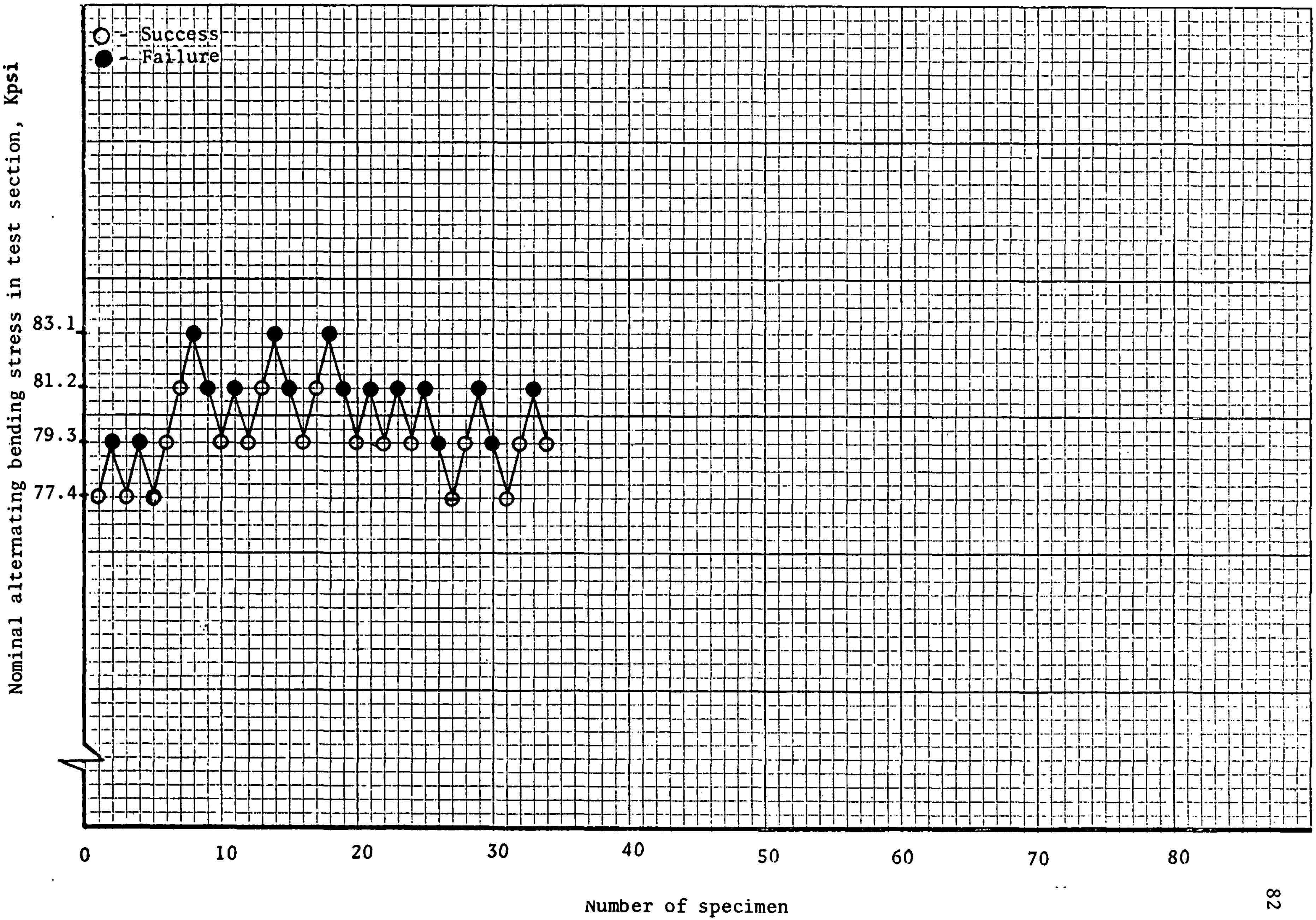
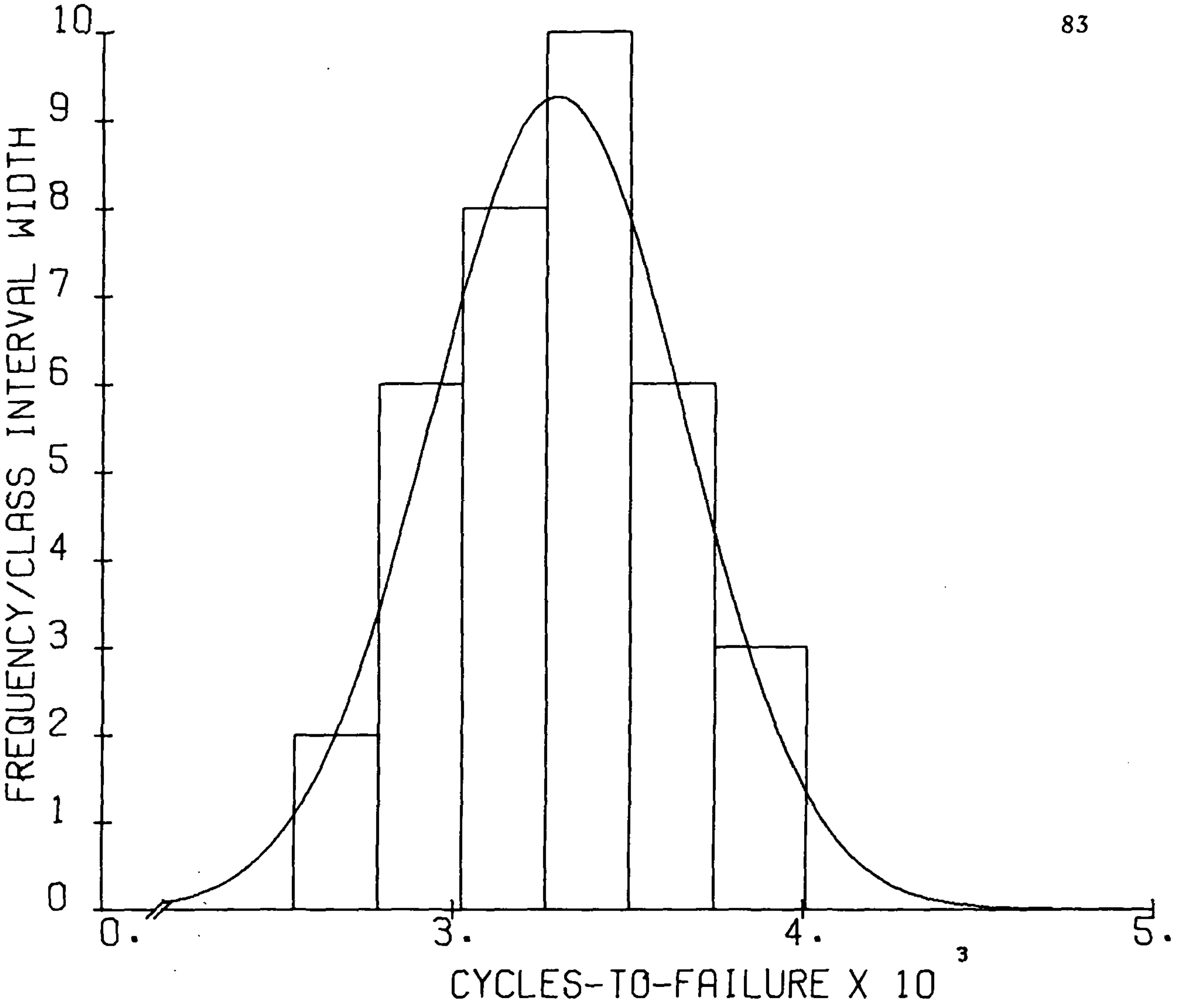


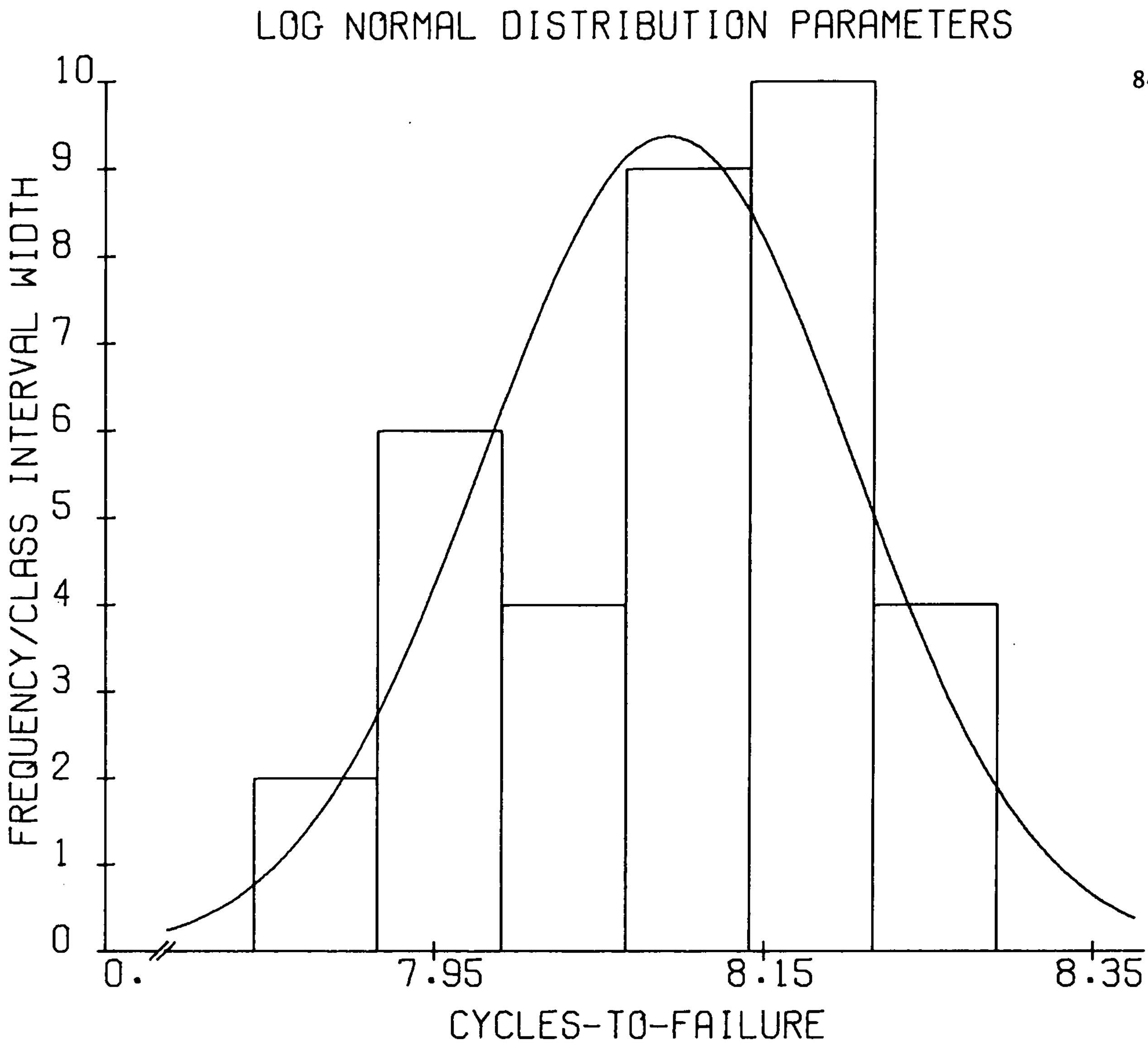
Fig. 23 Endurance strength data obtained by the staircase method in the Ann Arbor machine for the stress ratio of ∞ for AISI 4340 steel R_c 35/40 Phase II research. Specimens with no groove and a test section diameter of 0.2500 in.

NORMAL DISTRIBUTION PARAMETERS



MEAN VALUE:	3297.1	CYCLES
STANDARD DEVIATION:	361.7	CYCLES
KOLMOGOROV-SMIRNOV TEST:	0.094	
CHI-SQUARED TEST:	0.248	
SKEWNESS:	-0.163	
KURTOSIS:	2.317	

FIG. 24 CYCLES-TO-FAILURE DISTRIBUTION OF 35 GROOVED SPECIMENS FOR AN ALTERNATING STRESS LEVEL OF108,900 PSI AT A STRESS RATIO OF INFINITY AND NOMINAL GROOVE DIAMETER OF 0.491 INCHES.



MEAN VALUE:	8.095	CYCLES
STANDARD DEVIATION:	0.112	CYCLES
KOLMOGOROV-SMIRNOV TEST:	0.096	
CHI-SQUARED TEST:	2.844	
SKEWNESS:	-0.374	
KURTOSIS:	2.408	

FIG. 25 CYCLES-TO-FAILURE DISTRIBUTION OF 35 GROOVED SPECIMENS FOR AN ALTERNATING STRESS LEVEL OF108,900 PSI AT A STRESS RATIO OF INFINITY AND NOMINAL GROOVE DIAMETER OF 0.491 INCHES.

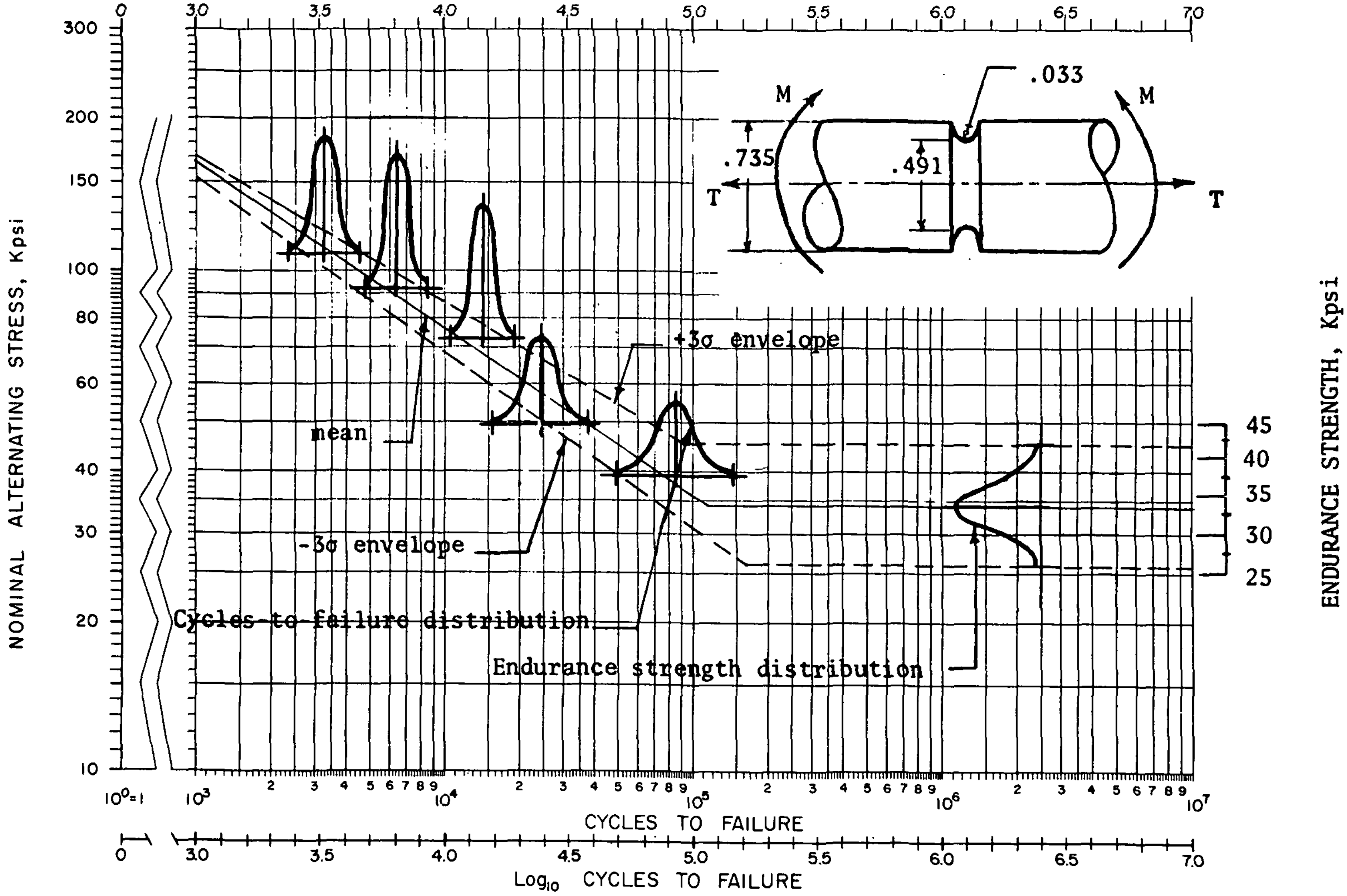


Fig. 26 Cycles-to-failure distributions at the stress ratio of ∞ for AISI 4340 R_c 35/40 Phase II grooved specimens.

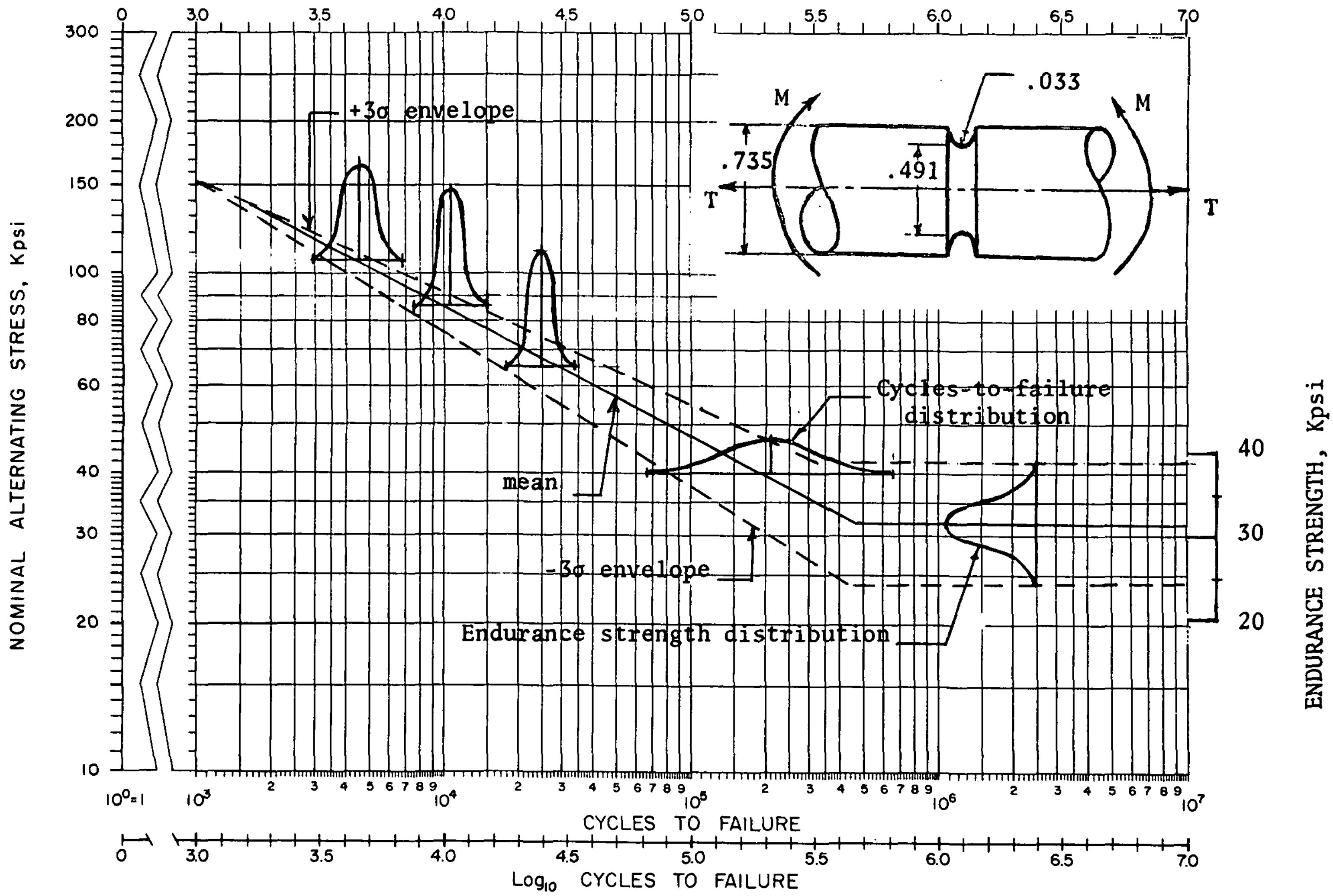


Fig. 27 Cycles-to-failure distribution at the stress ratio of 1.06 for AISI 4340 R_c 35/40 Phase II grooved specimens.

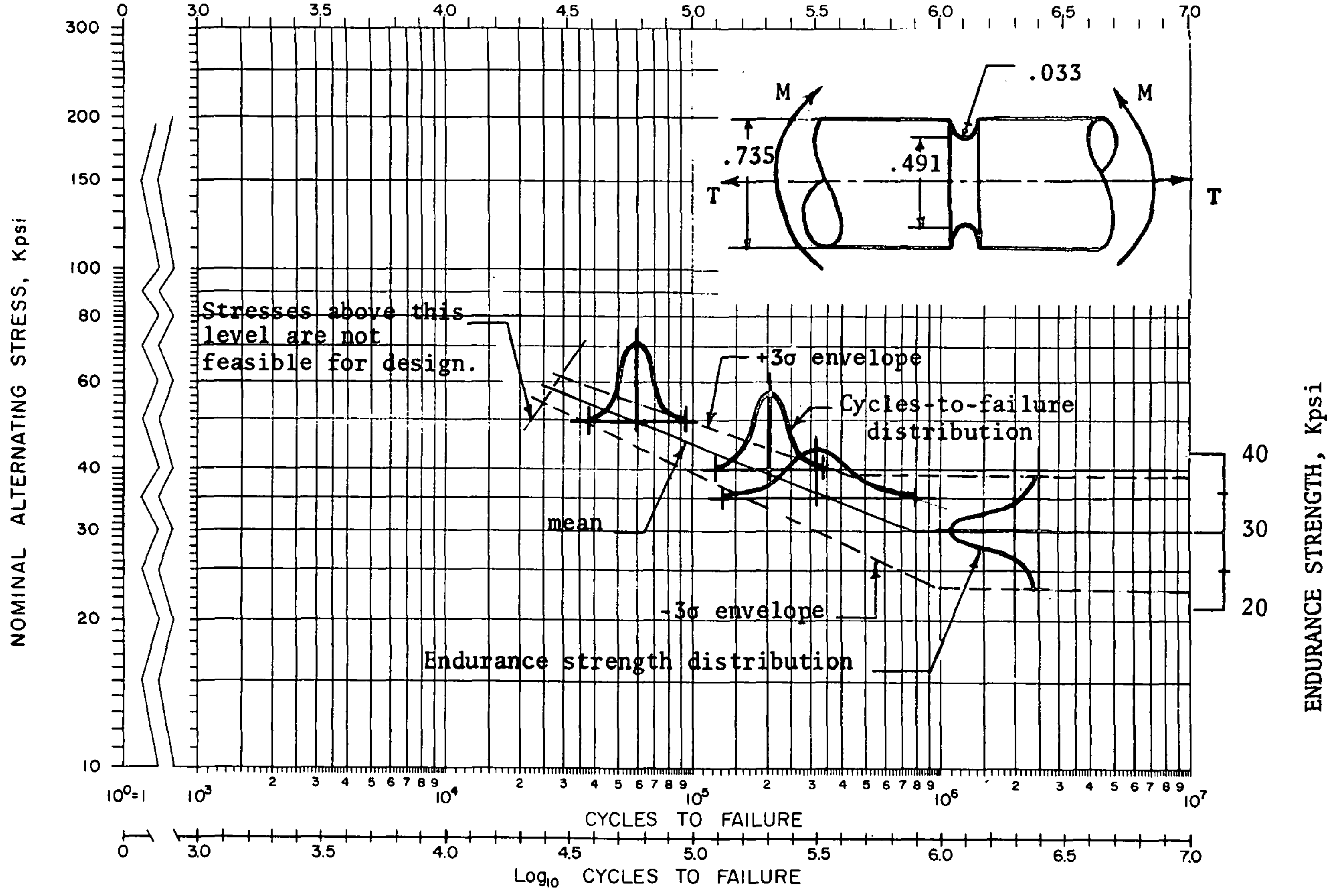


Fig. 28 Cycles-to-failure distribution at the stress ratio of 0.40 for AISI 4340 R_c 35/40 Phase II grooved specimens.

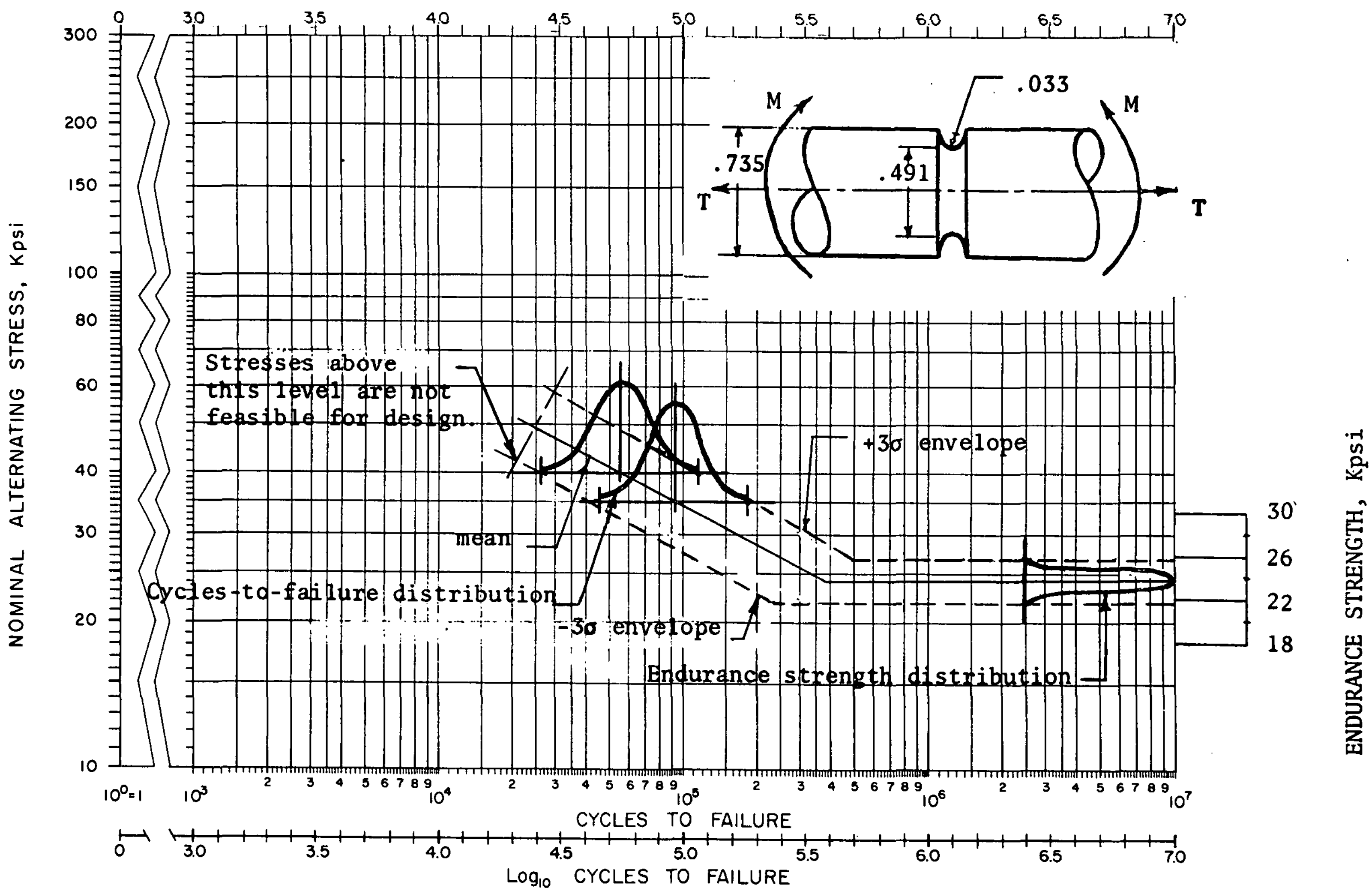


Fig. 29 Cycles-to-failure distribution at the stress ratio of 0.25 for AISI 4340 R_C 35/40 Phase II grooved specimens.

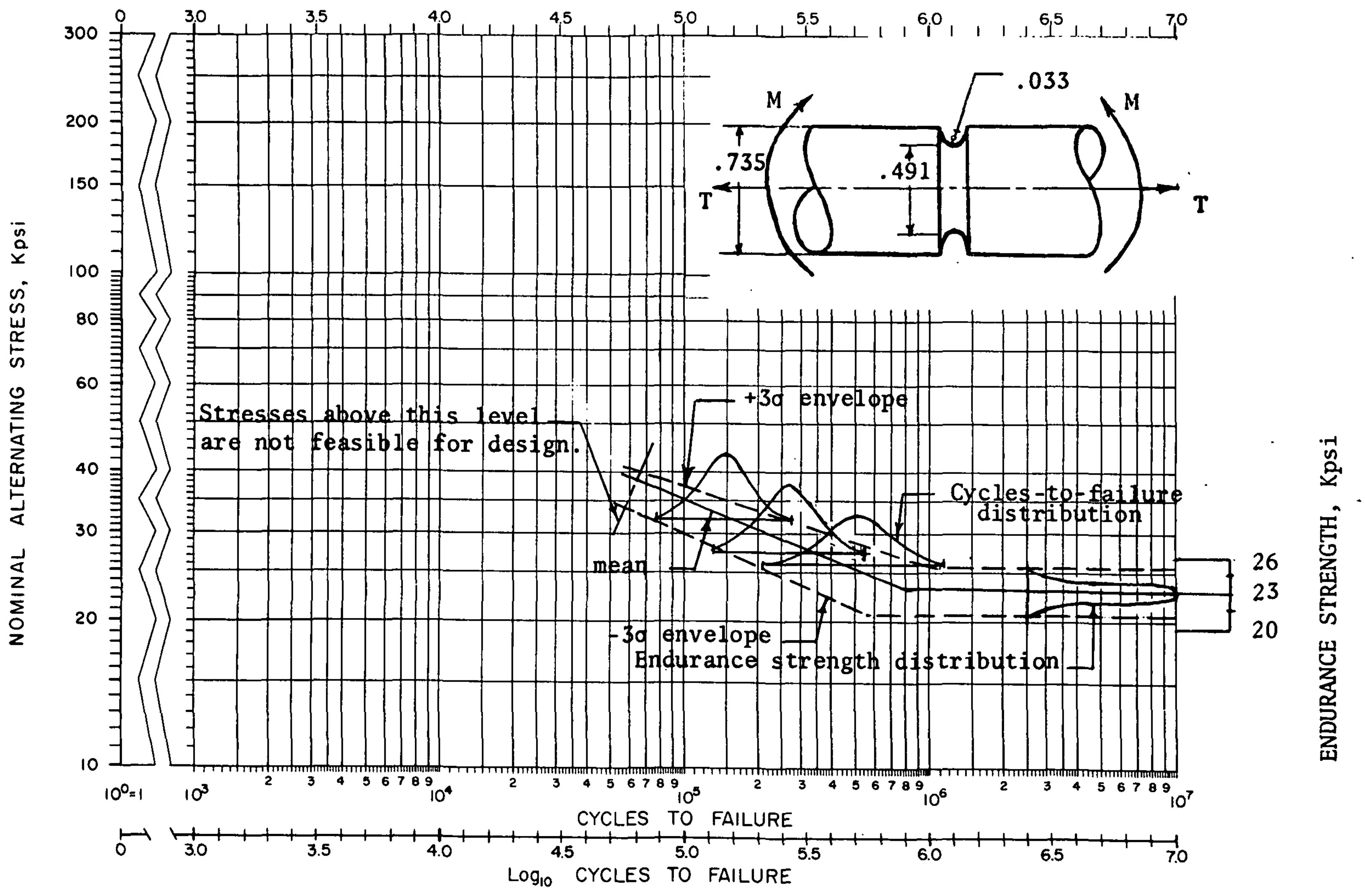


Fig. 30 Cycles-to-failure distribution at the stress ratio of 0.15 for AISI 4340 R_c 35/40 Phase II grooved specimens.

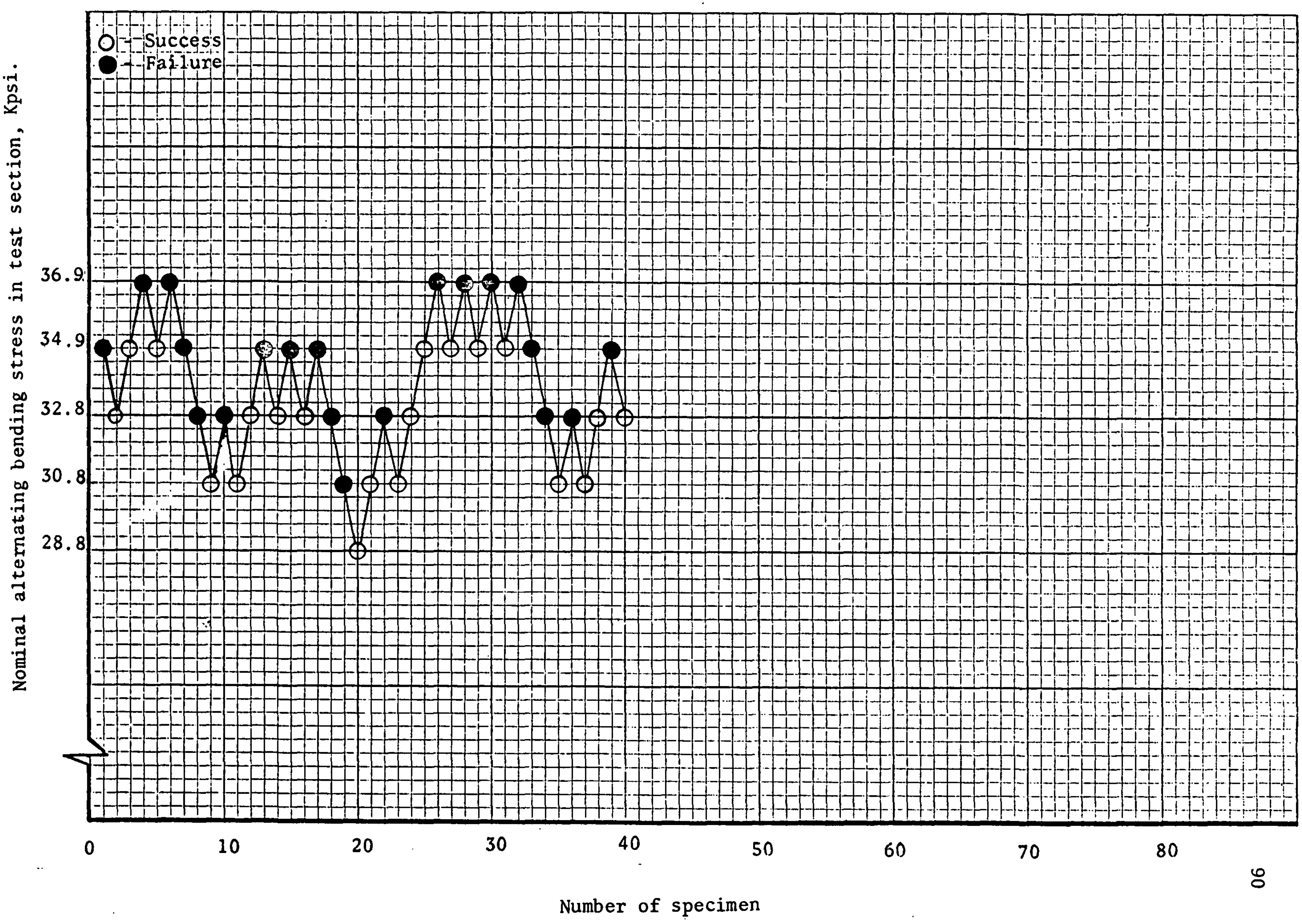


Fig. 31 Endurance strength data obtained by the staircase method for Phase II grooved specimens of AISI 4340 steel R_c 35/40 for stress ratio of ∞.

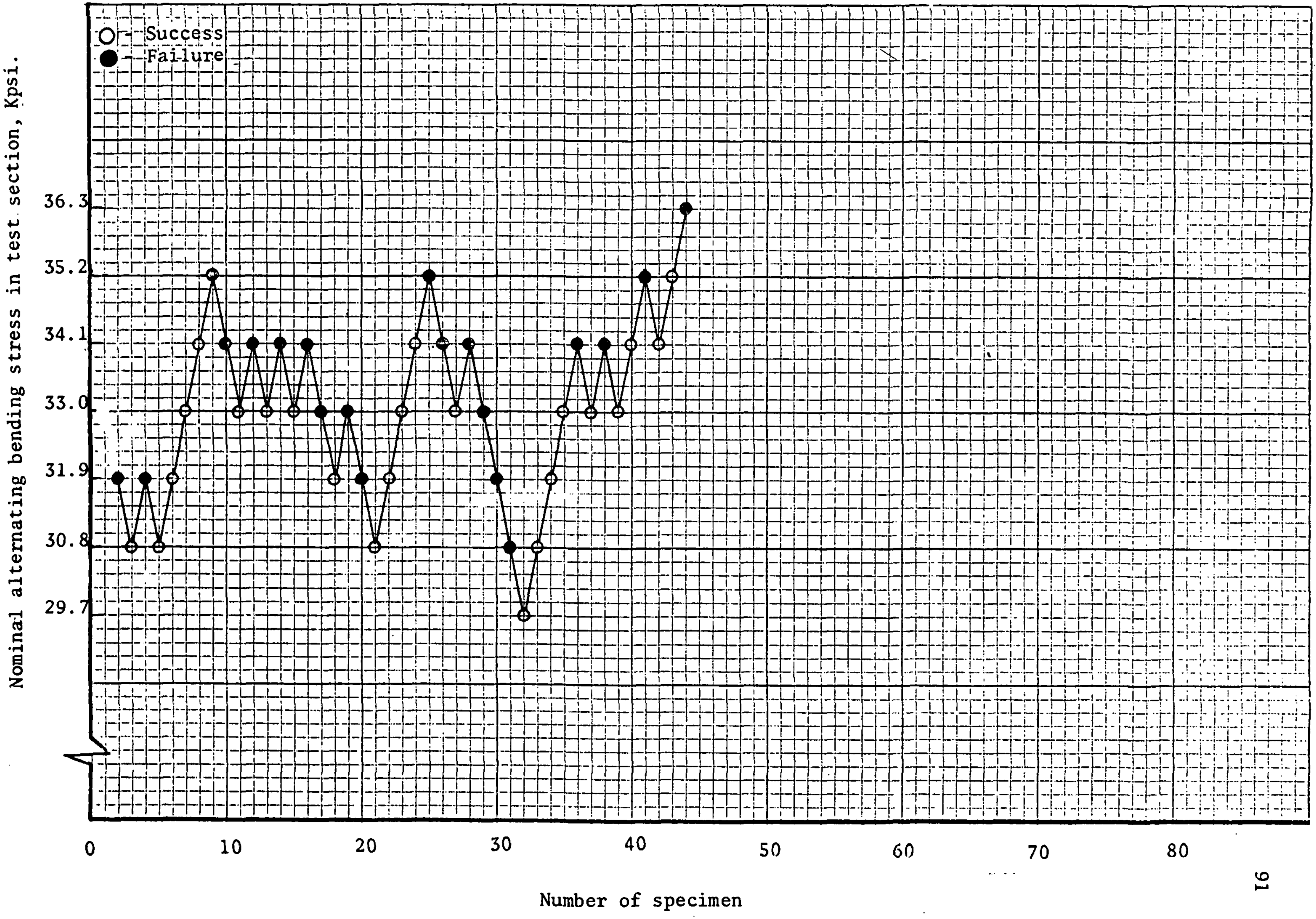


Fig. 32 Endurance strength data obtained by the staircase method for Phase II grooved specimens of AISI 4340 steel R_c 35/40 for stress ratio of 1.06.

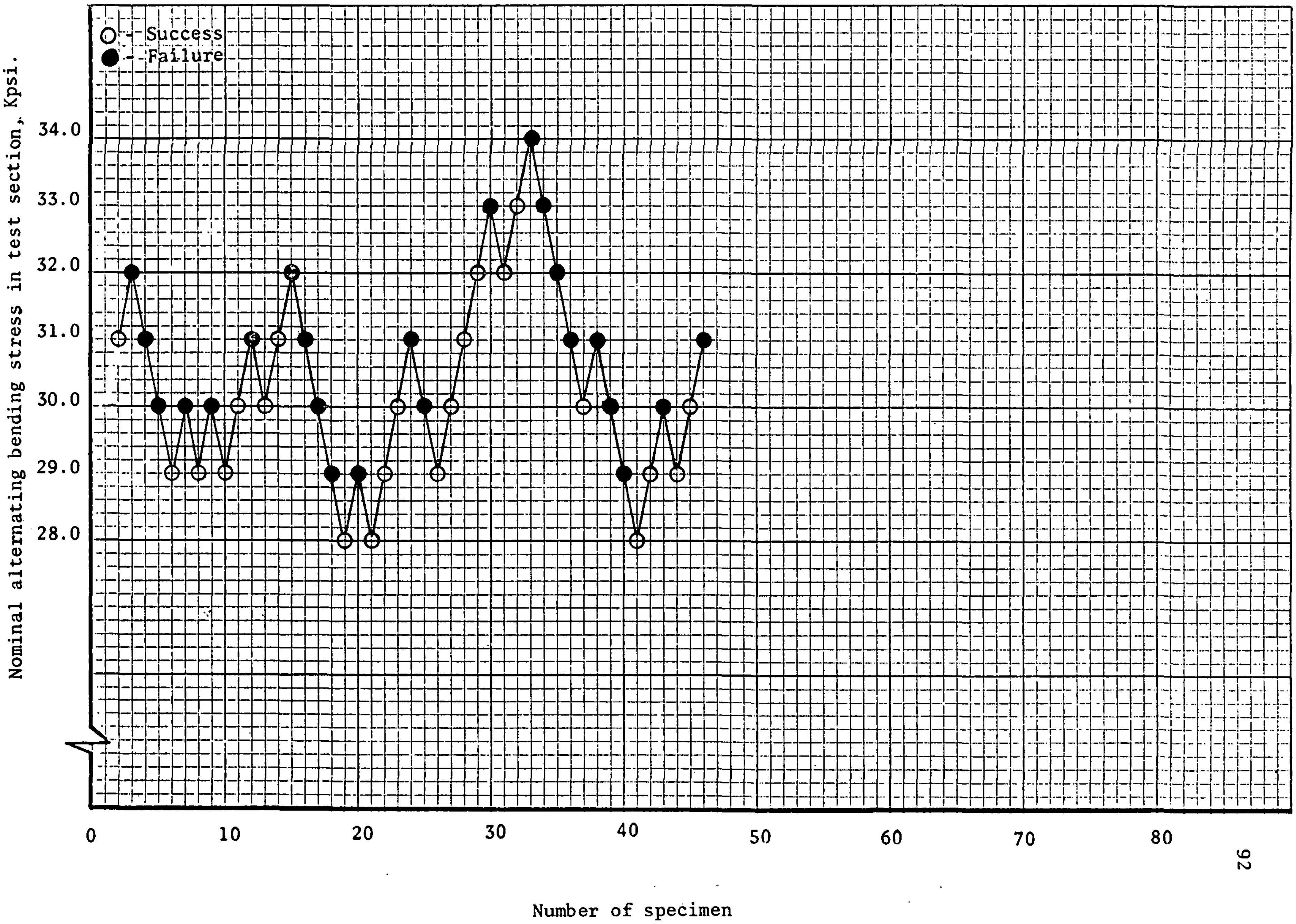


Fig. 33 Endurance strength data obtained by the staircase method for Phase II grooved specimens of AISI 4340 steel R_c 35/40 for stress ratio of 0.40.

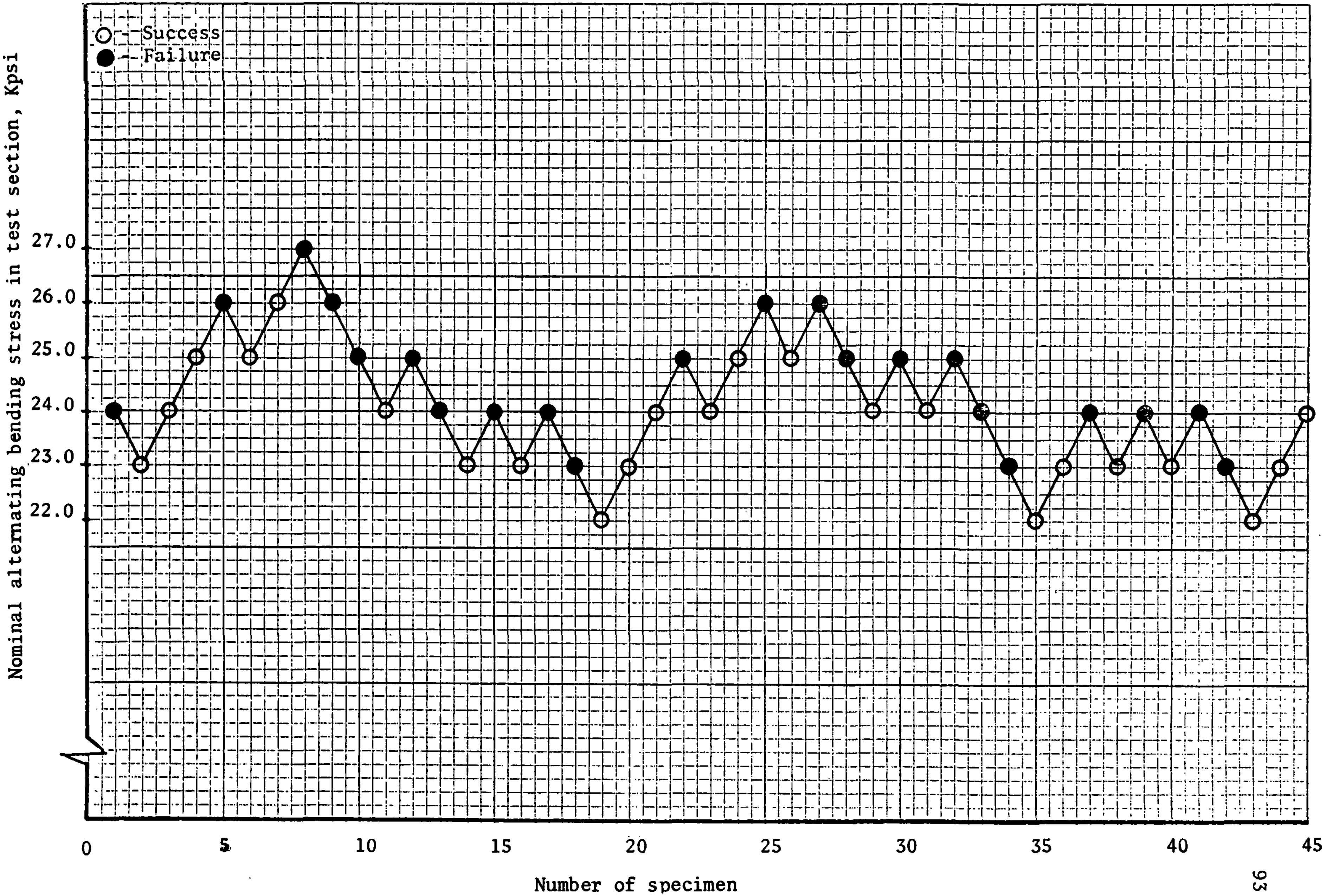


Fig. 34 Endurance strength data obtained by the staircase method with Phase II grooved specimens of AISI 4340 steel R_C 35/40 for stress ratio of 0.25.

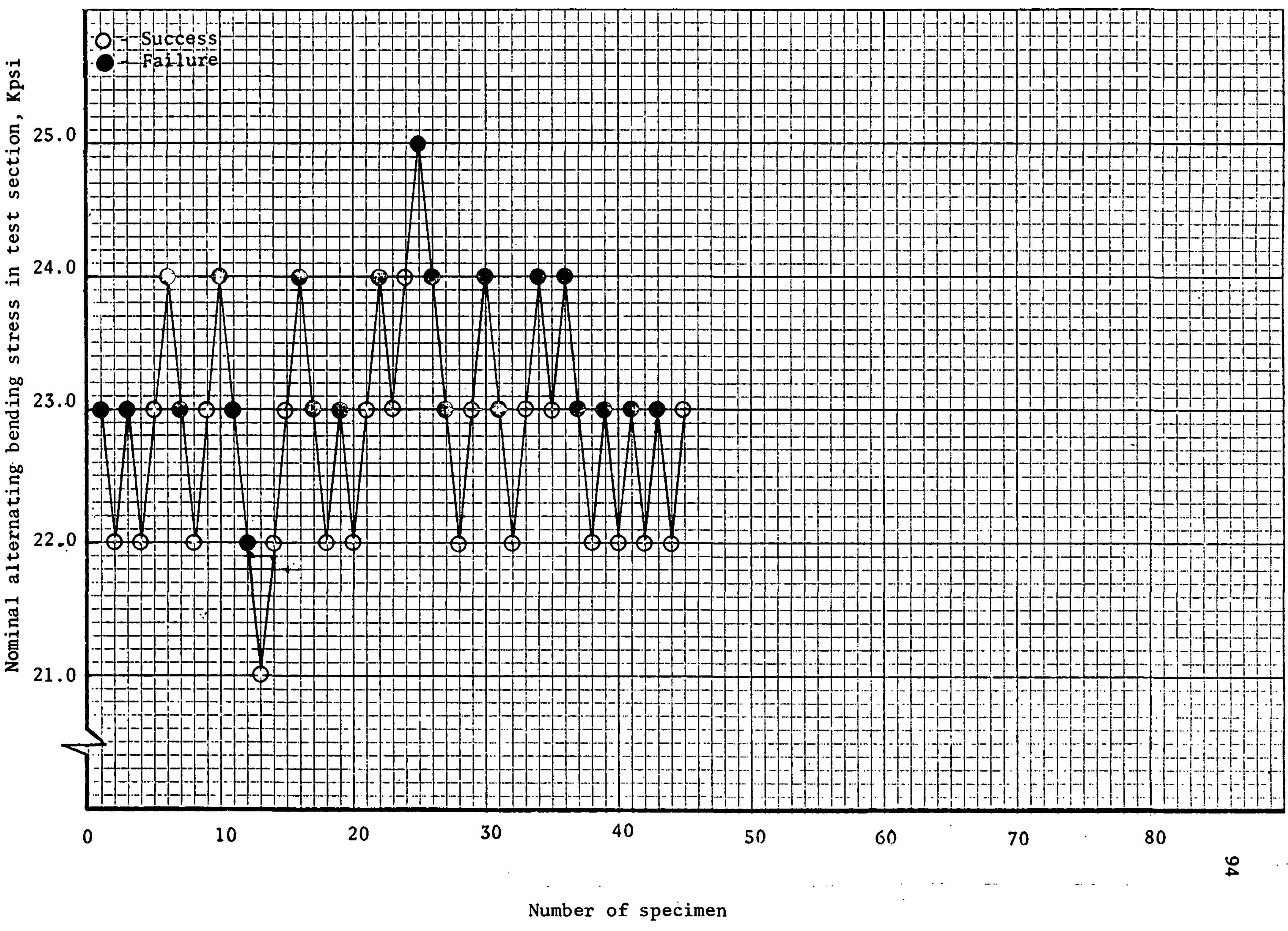


Fig. 35 Endurance strength data obtained by the staircase method for Phase II grooved specimens of AISI 4340 steel R_C 35/40 for stress ratio of 0.15.

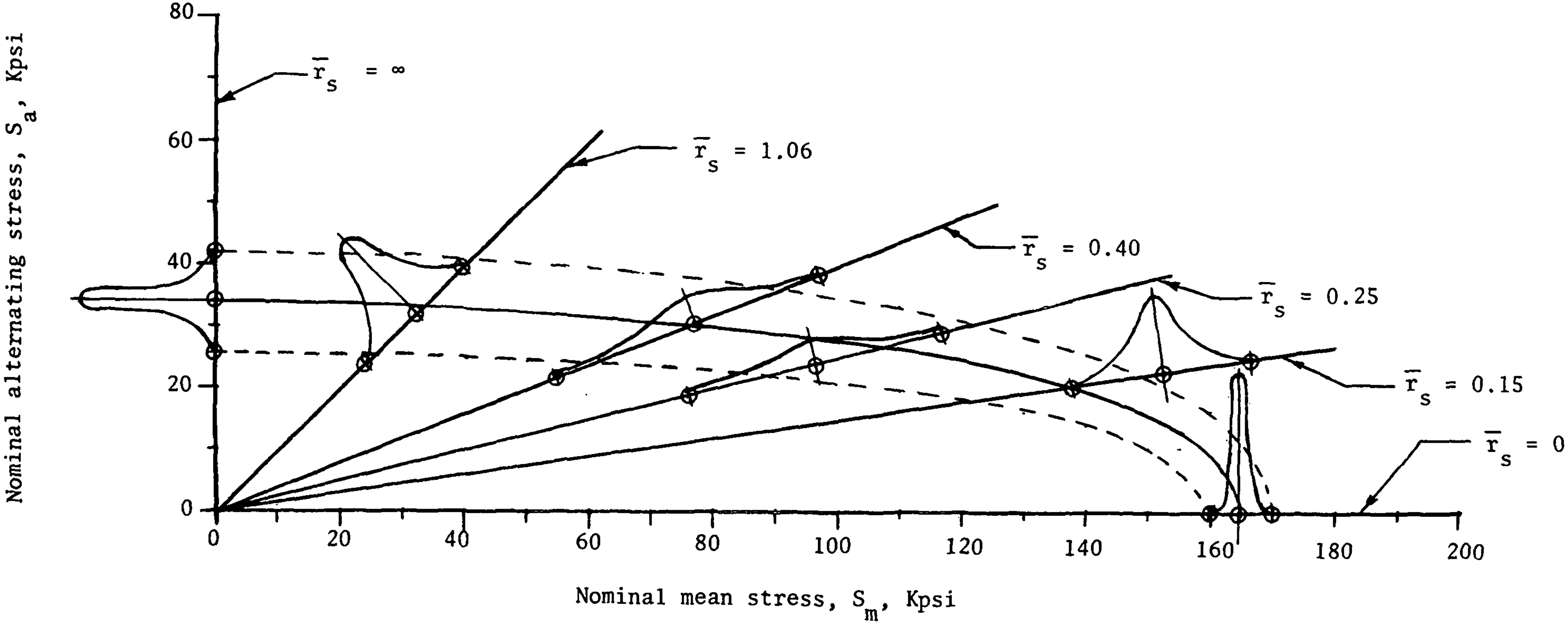
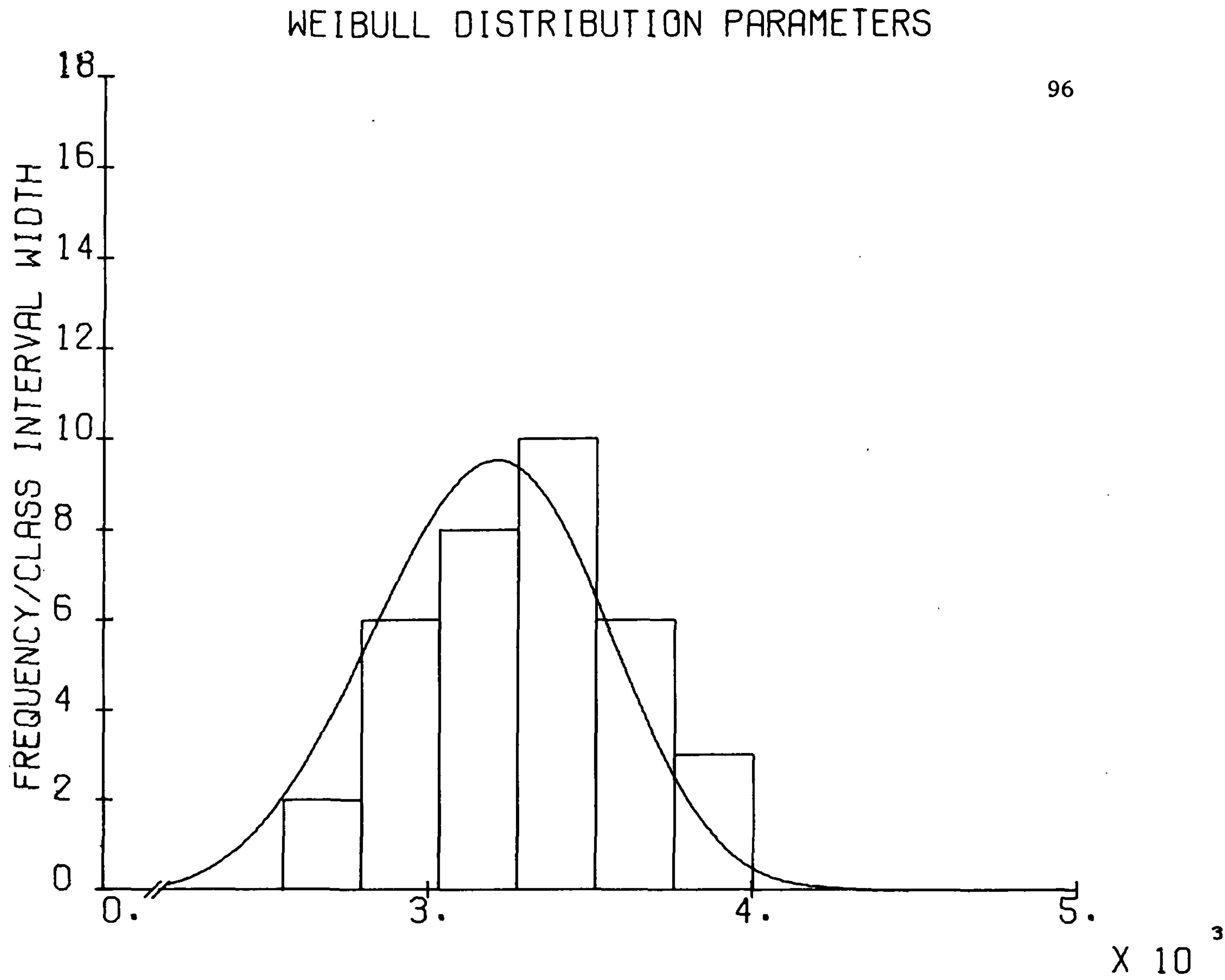


Fig. 36 Distributional Goodman strength diagram for 2.5×10^6 cycles of life Phase II results with AISI 4340 steel R_c 35/40 grooved specimens (See Table 30).



KOLMOGOROV-SMIRNOV TEST: 0.217
CHI-SQUARED TEST: 4.236
WEIBULL SLOPE (BETA): 3.853
MINIMUM LIFE (GAMMA): 1999
SCALE PARAMETER (ETA): 1306

FIG. 37 CYCLES-TO-FAILURE DISTRIBUTION OF 35 GROOVED SPECIMENS FOR AN ALTERNATING STRESS LEVEL OF 108,900 PSI AT A STRESS RATIO OF INFINITY AND NOMINAL GROOVES DIAMETER OF 0.491 INCHES.

TABLES

Table 1 Summary of nominal stresses applied to eighteen Phase I specimens for a nominal S_a = 65,000 psi and r_s = 0.44.

	Mean	Standard deviation
Bending Stress (psi), $\sigma_{xa} = S_a$	64,700	1,300
Shear Stress (psi), τ_{xym}	84,900	1,800
Stress Ratio = $\frac{S_a}{\sqrt{3}\,\tau_{xym}}$	0.44	0.01

Table 2 Estimates for cycles-to-failure distribution parameters for S_a = 65,000 psi and r_s = 0.44.

Distribution	Normal	Lognormal_e
Mean Cycles-to-Failure	83,700	11.278
Standard Deviation	26,350	0.374
Skewness	0.315	1.091
Kurtosis	2.408	3.747
K-S Test Maximum D Value	0.066	0.114
Allowable D Value at the 0.05 significance level	0.294	0.294

Table 3 Stress levels for cycles-to-failure tests in Phase I research for AISI 4340 steel, R_c 35/40 grooved specimens.

Stress ratio, r_s	Sample size	Nominal alternating bending stress, S_a		Nominal shear stress, τ_{xym}		Nominal normal mean stress, S_m	
Mean		Mean* psi	Std. dev.* psi	Mean* psi	Std. dev.* psi	Mean* psi	Std. dev.* psi
∞	12	144,200	1,500	0	0	0	0
	18	113,900	900	0	0	0	0
	18	97,900	2,700	0	0	0	0
	18	81,300	900	0	0	0	0
	18	73,100	1,800	0	0	0	0
3.5	12	151,000	3,800	24,800	800	43,100	1,400
	18	114,600	1,800	19,300	700	33,400	1,200
	18	83,200	1,200	13,700	700	23,800	1,200
	18	74,200	800	12,400	600	21,500	1,000
0.83	12	111,200	1,300	73,300	2,100	127,100	3,600
	18	91,500	6,700	66,300	3,500	115,000	6,700
	18	75,700	3,200	53,700	3,400	93,200	5,900
	18	65,200	3,900	46,800	1,300	81,200	2,300
0.44	18	68,900	1,400	89,800	800	155,600	1,400
	18	64,700	1,300	85,000	1,800	147,300	3,100
	18	59,600	700	78,400	1,400	135,800	2,400

* Rounded off to nearest 100 psi.
** $S_m = \sqrt{3}\,\tau_{xym}$ and $\sigma_{S_m} = \sqrt{3}\,\sigma_{\tau_{xym}}$ using the von Mises-Hencky failure theory.

Table 4 Normal distribution parameter estimates and the max D values for Phase I cycles-to-failure results.

Stress ratio	Nominal alternating bending stress Mean psi	Sample size	Normal distribution parameters				Max. D value*
			Mean cycles	Standard deviation cycles	Skew-ness	Kurtosis	
∞	144,200	12	2,773	532	-1.344	3.561	0.175
	113,900	18	9,029	1,024	-0.247	1.995	0.099
	97,900	18	22,171	3,815	-0.042	1.855	0.082
	81,300	18	77,977	12,549	0.873	3.060	0.184
	73,100	18	161,984	34,286	-0.616	2.201	0.174
3.5	151,000	12	1,451	275	-0.222	1.725	0.113
	114,600	18	6,203	1,010	0.797	3.397	0.111
	83,200	18	39,608	12,990	1.876	7.241	0.239
	74,200	18	74,212	19,531	0.147	2.057	0.132
0.83	111,200	12	6,554	1,005	0.893	2.493	0.296
	91,500	18	20,467	5,595	0.239	2.257	0.119
	75,700	18	61,028	11,132	-0.428	2.182	0.109
	65,200	18	127,577	21,227	0.171	1.942	0.141
0.44	68,900	18	53,573	15,470	1.667	4.833	0.272
	64,700	18	83,700	26,347	-0.315	2.408	0.067
	59,600	18	142,550	41,585	0.393	3.057	0.148

* Maximum D-Value from K-S Test

Table 5 Lognormal distribution parameter estimates and max D values for Phase I cycles-to-failure results.

Stress ratio	Nominal alternating bending stress Mean psi	Sample size	Lognormal distribution parameters				Max. D value*
			Mean $\log_e$	Standard deviation $\log_e$	Skew-ness	Kurtosis	
∞	144,200	12	7.907	0.228	-1.586	4.258	0.207
	113,900	18	9.102	0.116	-0.407	2.123	0.092
	97,900	18	9.992	0.176	-0.265	1.945	0.094
	81,300	18	11.253	0.154	0.575	2.697	0.159
	73,000	18	11.971	0.235	-0.906	2.707	0.198
3.5	151,000	12	7.263	0.198	-0.435	2.006	0.128
	114,600	18	8.721	0.157	0.403	2.869	0.086
	83,200	18	10.545	0.287	0.686	4.407	0.184
	74,200	18	11.180	0.274	-0.342	2.501	0.084
0.83	111,200	12	8.778	0.146	0.742	2.295	0.280
	91,500	18	9.890	0.281	-0.171	1.916	0.120
	75,700	18	11.002	0.195	-0.703	2.457	0.111
	65,200	18	11.743	0.167	0.030	1.769	0.147
0.44	68,900	18	10.857	0.248	1.210	3.899	0.214
	64,700	18	11.277	0.374	-1.091	3.747	0.115
	59,600	18	11.825	0.305	-0.457	3.182	0.118

* Maximum D-Value from K-S Test

Table 6 Endurance strength calculations for the stress ratio of 0. 45 using the staircase method at 2.5 x 10^6 cycles for AISI 4340 steel R_c 35/40 Phase I grooved specimens.

Nominal alternating stress psi	i	n_i Successes	in_i	i^2n_i
51,870	3	3	9	27
49,460	2	7	14	28
47,050	1	5	5	5
44,640	0	3	0	0
		N = 18	A = 28	B =60

d = stress increment = 2,410 psi

X_o = lowest stress level = 44,640 psi

$\overline{X}$ = mean (estimate)

$\overline{X} = X_o + d[A/N + 1/2] = 44,638 + 2,411 \left[\frac{28}{18} + \frac{1}{2} \right]$

$\overline{X} = 49,597$ psi $\cong$ 49,600* psi

s = standard deviation (estimate)

$s = 1.620\, d[(NB-A^2)/N^2 + 0.029] = 1.260\ (2,411) \left[\frac{18(60) - (28)^2}{(18)^2} + 0.029 \right]$

s = 3,681 psi $\cong$ 3,700* psi

---

* Rounded off to nearest 100 psi.

Table 7 . Endurance strength calculations for the stress ratio of ∞ using the staircase method at 2.5 x 10^6 cycles for AISI 4340 steel R_c 35/40 Phase I grooved specimens.

Alternating stress psi	i	n_i Successes	in_i	i^2n_i
61,857	3	1	3	9
58,543	2	2	4	8
55,229	1	10	10	10
51,915	0	2	0	0
		N = 15	A = 17	B = 27

d = stress increment = 3,314 psi

X_o = lowest stress level = 51,915 psi

$\overline{X}$ = mean (estimate)

$\overline{X} = X_o + d[A/N + 1/2] = 51,915 + 3,314 \left[\frac{17}{15} + \frac{1}{2} \right]$

$\overline{X}$ = 57,317 psi ≅ 57,300* psi

s = standard deviation (estimate)

$s = 1.620\, d[(NB-A^2)/N^2 + 0.029] = 1.620\,(3,314) \left[\frac{15(27) - (17)^2}{(15)^2} + 0.029 \right]$

s = 2,924 psi ≅ 2,900* psi

---

* Rounded off to nearest 100 psi.

Table 8 Endurance strength calculations for the stress ratio of 3.5 using the staircase method at 2.5×10^6 cycles for AISI 4340 steel R_c 35/40 Phase I grooved specimens.

Alternating stress psi	i	n_i Successes	in_i	i^2n_i
57,103	3	4	12	36
53,759	2	9	18	36
50,415	1	4	4	4
47,071	0	1	0	0
		N = 18	A = 34	B = 76

d = stress increment = 3,344 psi

X_o = lowest stress level = 47,071 psi

$\overline{X}$ = mean (estimate)

$\overline{X} = X_o + d[A/N + 1/2] = 47{,}071 + 3{,}344\ [\frac{34}{18} + \frac{1}{2}]$

$\overline{X} = 55{,}059$ psi $\cong$ 55,100* psi

s = standard deviation (estimate)

$s = 1.620\ d[(NB-A^2)/N^2 + 0.029] = 1.620\ (3{,}344)\ [\frac{18(76) - (34)^2}{(18)^2} + 0.029]$

s = 3,701 psi $\cong$ 3,700* psi

* Rounded off to nearest 100 psi.

Table 9 Endurance strength calculations for the stress ratio of 1.0 using the staircase method at 2.5×10^6 cycles for AISI 4340 steel R_c 35/40 Phase I grooved specimens.

Alternating stress psi	i	n_i Failures	in_i	$i^2 n_i$
60,168	5	1	5	25
58,707	4	2	8	32
57,246	3	6	18	54
55,785	2	6	12	24
54,324	1	2	2	2
52,863	0	1	0	0
		N = 18	A = 45	B = 137

d = stress increment = 1,461 psi

X_o = lowest stress level = 52,863 psi

$\overline{X}$ = mean (estimate)

$\overline{X} = X_o + d[A/N - 1/2] = 52{,}863 + 1{,}461 \left[\frac{45}{18} - \frac{1}{2} \right]$

$\overline{X}$ = 55,785 psi ≅ 55,800* psi

s = standard deviation (estimate)

$s = 1.620\, d[(NB-A^2)/N^2 + 0.029] = 1.620\,(1{,}461) \left[\frac{18(137) - (45)^2}{(18)^2} + 0.029 \right]$

s = 3,290 psi ≅ 3,300* psi

* Rounded off to nearest 100 psi.

Table 10 Summary of Phase I endurance strength results at 2.5 x 10^6 cycles of life for AISI 4340 steel R_c 35/40 grooved specimens for distributional Goodman diagram.

Stress ratio, r_s	Nominal alternating bending stress, S_a		Nominal normal mean stress**, S_m		Nominal combined stress vector***, S_v	
	Mean*, $\bar{S}_a$ psi	Std. dev.* σ_{S_a} psi	Mean*, $\bar{S}_m$ psi	Std. dev.*, σ_{S_m} psi	Mean*, $\bar{S}_v$ psi	Std. dev.*, σ_{S_v} psi
∞	57,300	2,900	0	0	57,300	2,900
3.5	55,100	3,700	15,700	1,100	57,300	3,900
1.0	55,800	3,300	56,800	3,300	78,900	4,700
0.45	49,600	3,700	110,200	8,200	120,900	9,000

* Rounded off to nearest 100 psi.

** $\bar{S}_m = \frac{\bar{S}_a}{\bar{r}_s}$ and $\sigma_{S_m} = \frac{1}{\bar{\bar{r}}_s} \sigma_{S_a}$

*** $\bar{S}_v = \bar{S}_a \left(1 + \frac{1}{\bar{r}_s^2}\right)^{1/2}$ and $\sigma_{S_v} = \sigma_{S_a} \left(1 + \frac{1}{\bar{r}_s^2}\right)^{1/2}$

Table 11 Diameter and surface hardness at test section of AISI 4340 steel Phase II ungrooved specimens

Specimen no.	Diameter in.	Hardness Rockwell C
1	0.4977	37.6
2	0.4980	36.3
3	0.4981	37.0
4	0.4979	36.5
5	0.4980	37.5
6	0.4967	38.3
7	0.4986	36.2
8	0.4981	37.0
9	0.4979	37.3
10	0.4985	36.5
11	0.4981	36.5
12	0.4983	36.8
13	0.4974	36.3
14	0.4982	36.3
15	0.4983	37.0
16	0.4980	36.5
17	0.4981	37.3
18	0.4982	36.3
19	0.4993	37.4
20	0.4988	37.6
21	0.4986	39.2
22	0.4993	37.5
23	0.4980	36.8
24	0.4992	36.7
25	0.4996	38.3
26	0.4979	37.4
27	0.5000	37.6
28	0.5000	37.0
29	0.4981	36.5
30	0.4978	37.0
31	0.4978	37.6
32	0.4983	37.5
33	0.4977	37.3
34	0.4984	37.0
35	0.4978	35.0

<u>Diameter</u>		<u>Hardness</u>	
Mean	= 0.4983 in.	Mean	= 37.05 R_c
Standard deviation	= 0.0006 in.	Standard deviation	= 0.75 R_c

Table 12 Diameter at the base of groove, groove radius, and surface hardness of AISI 4340 steel Phase II grooved specimens.

Specimen no.	Diameter in.	Radius in.	Surface hardness Rockwell C
1	0.4928	0.35	38.1
2	0.4926	0.35	37.6
3	0.4909	0.35	37.6
4	0.4922	0.35	38.1
5	0.4953	0.35	37.3
6	0.4966	0.35	37.5
7	0.4966	0.35	36.3
8	0.4981	0.35	37.6
9	0.4936	0.35	38.0
10	0.4937	0.35	37.4
11	0.4972	0.35	37.5
12	0.4938	0.35	36.6
13	0.4947	0.35	37.6
14	0.4935	0.35	37.5
15	0.4973	0.35	36.5
16	0.4943	0.35	37.1
17	0.4907	0.35	37.2
18	0.4948	0.35	37.4
19	0.4952	0.35	37.5
20	0.4941	0.35	37.0
21	0.4969	0.35	38.3
22	0.4940	0.35	36.5
23	0.4938	0.35	37.5
24	0.4930	0.35	37.3
25	0.4934	0.35	37.4
26	0.4931	0.35	38.0
27	0.4937	0.35	36.3
28	0.4950	0.35	37.3
29	0.4934	0.35	37.2
30	0.4924	0.35	37.0
31	0.4932	0.35	36.5
32	0.4927	0.35	38.1
33	0.4939	0.35	36.9
34	0.4945	0.35	37.8
35	0.4933	0.35	37.0

Diameter		Hardness	
Mean	= 0.4939 in.	Mean	= 37.29 R_c
Standard deviation	= 0.0021 in.	Standard deviation	= 0.53 R_c

Table 13 Hardness measurements at Section A of Phase II grooved specimens in R_c units see Fig. 17 for locations.

Specimen No.	Location No. 1	2	3	4	5	6	7	8	Axis
33		34.5	36.5	36.5	37.0	36.5	36.5		0^o
		36.0	36.0	37.0	37.0	37.0	36.5		0^o
		35.5	35.5	36.0	37.0	36.5	36.0		90^o
		36.0	36.5	37.5	37.0	36.0	37.0		90^o
	$\overline{X}$ =	35.5	36.2	36.8	37.0	36.5	36.5		
	$\overline{\overline{X}}$ =	36.4 R_c							
34	36.0	37.0	36.5	37.5	37.5	37.5	37.0	36.5	0^o
	36.5	36.5	36.5	36.0	36.5	36.5	36.0	34.5	0^o
	37.0	36.0	36.5	37.0	37.0	37.0	36.5	36.5	90^o
	36.0	37.0	37.5	38.0	38.0	37.5	37.0	36.5	90^o
	$\overline{X}$ = 36.4	36.7	36.8	37.2	37.3	37.2	36.7	36.0	
	$\overline{\overline{X}}$ = 36.8 R_c								
35	35.0	35.0	37.0	38.0	37.0	37.5	37.0	37.5	0^o
	37.0	37.5	37.0	38.0	37.5	37.5	38.0	37.0	0^o
	34.0	35.5	36.5	37.5	37.0	36.0	36.5	36.0	90^o
	36.0	35.5	37.5	38.0	37.0	37.0	36.5	36.5	90^o
	$\overline{X}$ = 36.5	35.9	37.0	37.9	37.1	37.0	37.0	36.9	
	$\overline{\overline{X}}$ = 36.9 R_c								

Table 14 Hardness measurements for Phase II specimens at Section A shown in Fig. 18 in R_c units

Ungrooved Specimens				Grooved Specimens			
Specimen no.	R_c units	$\bar{X}$	$\bar{\bar{X}}$	Specimen no.	R_c units	$\bar{X}$	$\bar{\bar{X}}$
31	C = 36.5 36.5 37.0	36.7		33	C = 36.5 37.5 37.0	37.0	
	I = 36.0 36.0 35.0	35.7			I = 36.0 38.0 37.0	37.0	
	O = 34.5 35.0 36.5	35.3	35.9		O = 36.0 36.0 36.0	36.0	36.7
33	C = 37.5 38.0 38.0	37.8		34	C = 35.0 36.5 36.0	36.0	
	I = 37.0 38.0 37.5	37.5			I = 36.0 35.0 36.0	35.7	
	O = 36.5 36.0 35.0	36.0	37.1		O = 37.0 35.0 35.0	35.7	35.8
34	C = 33.5 35.5 34.5	34.5		35	C = 37.0 37.5 37.0	37.2	
	I = 37.0 36.0 37.0	36.5			I = 35.5 37.0 36.0	36.2	
	O = 35.5 35.0 36.0	35.5	35.5		O = 35.0 35.5 35.0	35.2	36.2

Table 15 . Tensile test data for Phase II AISI 4340 steel R_c 35/40 ungrooved specimens.

Specimen number	Original diameter in.	Original cross section area in.^{2}	Yield load klb	Yield stress psi	Ultimate load klb	Ultimate stress psi	Breaking load klb	Breaking diameter in.	Breaking area in.^{2}	Breaking stress psi
1	0.4977	0.1945	30.8	158,300	32.4	166,500	21.5	0.3249	0.0829	259,300
2	0.4980	0.1948	29.9	153,500	32.1	164,800	21.5	0.3248	0.0829	259,500
3	0.4981	0.1948	30.3	155,500	32.3	165,800	21.6	0.3276	0.0843	256,300
4	0.4979	0.1947	29.4	151,000	31.4	161,300	20.3	0.3161	0.0785	258,700
5	0.4980	0.1948	30.4	156,100	32.0	164,300	21.2	0.3221	0.0815	260,200
6	0.4967	0.1938	30.5	157,400	32.5	167,700	21.8	0.3281	0.0846	257,300
7	0.4986	0.1952	30.3	155,200	32.4	166,000	22.0	0.3296	0.0853	257,900
8	0.4981	0.1949	30.1	154,500	32.2	165,200	21.8	0.3311	0.0861	252,600
9	0.4979	0.1947	30.0	154,100	31.8	163,300	21.0	0.3195	0.0802	261,900
10	0.4985	0.1952	30.1	154,200	32.0	164,000	22.1	-	-	-
11	0.4981	0.1949	29.5	151,400	31.4	161,100	20.8	0.3194	0.0801	259,600
12	0.4983	0.1950	30.1	154,300	32.0	164,100	21.5	0.3231	0.0815	263,900
13	0.4974	0.1943	30.2	155,400	32.0	164,700	21.3	0.3227	0.0818	259,800
14	0.4982	0.1949	29.9	153,400	31.7	162,600	20.6	0.3188	0.0798	258,100
15	0.4983	0.1950	30.7	152,400	32.3	165,600	21.6	0.3228	0.0818	263,300
16	0.4980	0.1948	30.4	156,100	32.2	165,300	21.3	0.3232	0.0820	259,000
17	0.4981	0.1949	30.7	157,600	32.2	165,300	21.1	0.3224	0.0816	258,500
18	0.4982	0.1949	30.1	154,400	32.2	165,200	22.1	0.3370	0.0892	247,800
19	0.4993	0.1958	30.8	157,300	32.5	165,000	21.4	0.3237	0.0823	260,000
20	0.4988	0.1954	30.8	157,600	32.5	166,300	21.6	0.3248	0.0829	260,700
21	0.4986	0.1952	30.5	156,200	32.4	166,000	21.4	0.3203	0.3203	265,600

Table 15 Tensile test data for Phase II AISI 4340 steel R_c 35/40 ungrooved specimens (Cont'd.)

Specimen number	Original diameter in.	Original cross Section area in.^{2}	Yield load klb	Yield stress psi	Ultimate Load klb	Ultimate stress psi	Breaking Load klb	Breaking diameter in.	Breaking area in.^{2}	Breaking stress psi
22	0.4993	0.1958	30.6	156,300	32.6	166,500	22.1	0.3305	0.0858	257,000
23	0.4980	0.1948	30.5	156,600	32.3	165,800	21.8	0.3319	0.0865	252,000
24	0.4992	0.1957	30.6	156,300	32.4	165,500	21.5	0.3264	0.0837	257,000
25	0.4996	0.1960	30.6	156,100	32.4	165,300	21.2	0.3220	0.0814	260,300
26	0.4979	0.1947	30.6	157,200	32.3	165,900	21.6	0.3228	0.0818	263,900
27	0.5000	0.1963	30.8	156,900	32.7	166,500	22.0	0.3309	0.0860	255,800
28	0.5000	0.1963	30.6	155,900	32.4	165,000	21.5	0.3231	0.0820	262,200
29	0.4981	0.1949	30.1	154,500	32.0	164,200	20.8	0.3158	0.0783	265,600
30	0.4978	0.1946	29.7	152,600	31.6	162,400	20.7	0.3183	0.0796	260,100
31	0.4978	0.1946	30.5	156,700	32.4	166,500	21.5	0.3252	0.8310	258,900
32	0.4983	0.1950	30.3	155,400	32.4	166,100	21.5	0.3116	0.0763	281,900
33	0.4977	0.1945	30.5	156,800	32.4	166,500	21.3	0.3239	0.0824	258,500
34	0.4984	0.1951	30.0	153,800	32.1	164,500	20.8	0.3164	0.0786	263,900
35	0.4978	0.1946	30.5	156,700	32.5	167,000	-	-	-	-

Summary of static strength for ungrooved specimens

Strength	Mean psi	Standard deviation psi
Yield	155,500	1,800
Ultimate	165,100	1,500
Breaking	260,900	8,200

Table 16 Elongation test data for Phase II AISI 4340 steel R_c 35/40 ungrooved specimens.

Specimen no.	Elongation 2-in. gage length, in.	Elongation %
1	2.365	18.25
2	2.362	18.10
3	2.365	18.25
4	2.380	19.00
5	2.356	17.80
6	2.357	17.85
7	2.349	17.45
8	2.368	18.40
9	2.362	18.10
10	2.357	17.85
11	2.357	17.85
12	2.360	18.00
13	2.361	18.05
14	2.361	18.05
15	2.359	17.45
16	2.353	17.65
17	2.354	17.70
18	2.347	17.35
19	2.360	18.00
20	2.361	18.05
21	2.360	18.00
22	2.347	17.35
23	2.340	17.00
24	2.360	18.00
25	2.353	17.65
26	2.361	18.05
27	2.358	17.90
28	2.365	18.25
29	2.365	18.25
30	2.361	18.05
31	2.356	17.80
32	2.356	17.80
33	2.356	17.80
34	2.361	18.05

Table 17 Tensile test data for AISI 4340 steel R_c 35/40 grooved specimens.

Specimen number	Original diameter in.	Original cross section area, in.^{2}	Ultimate load lb	Ultimate stress psi
1	0.4928	0.1906	52,500	275,400
2	0.4926	0.1906	51,950	272,600
3	0.4909	0.1892	51,750	273,500
4	0.4922	0.1903	51,900	272,700
5	0.4953	0.1927	52,200	270,900
6	0.4966	0.1937	51,400	265,400
7	0.4966	0.1937	51,300	264,800
8	0.4981	0.1949	52,200	267,800
9	0.4936	0.1913	52,000	271,800
10	0.4937	0.1914	51,500	269,000
11	0.4972	0.1942	52,300	269,300
12	0.4938	0.1915	51,600	269,500
13	0.4947	0.1922	52,000	270,600
14	0.4935	0.1912	51,600	269,900
15	0.4973	0.1942	51,900	267,300
16	0.4943	0.1919	51,450	268,100
17	0.4907	0.1891	51,300	271,300
18	0.4948	0.1923	51,600	268,300
19	0.4952	0.1926	52,100	270,500
20	0.4941	0.1917	51,600	269,200
21	0.4969	0.1939	52,500	270,800
22	0.4940	0.1916	51,250	267,500
23	0.4938	0.1914	51,150	267,200
24	0.4930	0.1909	51,050	267,400
25	0.4934	0.1912	51,800	270,900
26	0.4931	0.1909	51,650	270,600
27	0.4937	0.1914	50,900	265,900
28	0.4950	0.1924	51,550	267,900
29	0.4934	0.1912	51,500	269,400
30	0.4924	0.1905	51,000	267,700
31	0.4923	0.1904	49,600	260,500
32	0.4927	0.1907	51,250	268,700

<u>Summary of static strength results grooved specimens.</u>
<u>Ultimate strength</u>

Mean = 269,000 psi

Standard deviation = 2,800 psi

Table 18 Summary of tensile strength parameters for Phase I and Phase II specimens.

Ungrooved	Mean	Standard deviation
Yield	psi	psi
Phase I	171,000	2,800
Phase II	155,500	1,800
Ultimate		
Phase I	177,900	2,600
Phase II	165,100	1,500
Breaking		
Phase I	254,800	4,400
Phase II	261,000	8,200
Grooved		
Ultimate		
Phase I	255,300	2,700
Phase II	269,100	2,800

Table 19 Calibration coefficients and speed for each research machine and for Mode 5 operation*.

Machine no.	K_{BGR}	K_{GR-TH}	K_{T}	$K_{T/B}$	$K_{B/T}$	Machine speed rpm
1	1.3081	0.0149	0.8753	-0.0462	0.0477	1779
2	1.3081	0.0145	0.8950	0.0463	0.0555	1775
3	1.3081	0.0170	0.8748	0.0542	0.0645	1778

*Mode 5 operation refers to the computer program code which identifies the proper calibration coefficients to be used with each set of machine data obtained during a specific calendar period. Mode 5 calibration coefficients should be used for all data obtained after June 1, 1971 until the next calibration.

Table 20 Endurance strength distribution parameters calculations for 0.3150 in. Phase I research AISI 4340 steel R_c 35/40 ungrooved specimens for stress ratio of ∞.

Alternating stress psi	i	n_i Failures	in_i	i^2n_i
85,000	4	1	4	16
83,100	3	5	15	45
81,200	2	3	6	12
79,300	1	4	4	4
77,400	0	0	0	0
		N = 13	A = 29	B = 77

d = stress increment = 1,900 psi

X_o = lowest stress level = 77,400 psi

$\overline{X}$ = mean (estimate)

$\overline{X} = X_o + d[A/N - 1/2] = 77,400 + 1,900 \left[\frac{29}{13} - \frac{1}{2}\right]$

$\overline{X}$ = 80,725 psi ≅ 80,700* psi

s = standard deviation (estimate)

$s = 1.620\, d[(NB-A^2)/N^2 + 0.029] = 1.620\,(1,900) \left[\frac{13(77) - (29)^2}{(13)^2} + 0.029\right]$

s = 3,040 psi ≅ 3,000* psi

* Rounded off to nearest 100 psi.

Table 21 Endurance strength distribution parameters calculations for 0..2500 in. Phase II research AISI 4340 steel, R_c 35/40 ungrooved specimens for stress ratio of ∞.

Alternating stress psi	i	n_i Failures	in_i	i^2n_i
83,100	3	3	9	27
81,200	2	9	18	36
79,300	1	4	4	4
77,400	0	0	0	0
		N = 16	A = 31	B = 67

d = stress increment = 1,900 psi

X_o = lowest stress level = 77,400 psi

$\overline{X}$ = mean (estimate)

$$\overline{X} = X_o + d[A/N - 1/2] = 77,400 + 1,900 \left[\frac{31}{16} - \frac{1}{2}\right]$$

$\overline{X}$ = 80,230 psi ≅ 80,200* psi

s = standard deviation (estimate)

$$s = 1.620\, d[(NB-A^2)/N^2 + 0.029] = 1.620\ (1,900) \left[\frac{16(67) - (31)^2}{(16)^2} + 0.029\right]$$

s = 1,425 psi ≅ 1,400* psi

* Rounded off to nearest 100 psi.

Table 22 Stress levels for cycles-to-failure tests in Phase II research for AISI 4340 steel R_c 35/40 grooved specimens.

Average Stress Ratio, r_s	Sample Size	Nominal alternating bending stress, S_a Mean* psi	Std. dev.* psi	Nominal shear stress, τ_{xym} Mean psi	Std. dev. psi	Nominal normal mean stress, S_m** Mean psi	Std. dev. psi
∞	35	108,900	1,300	0	0	0	0
	35	92,100	700	0	0	0	0
	35	73,600	800	0	0	0	0
	35	49,400	300	0	0	0	0
	35	39,300	400	0	0	0	0
1.06	35	105,700	1,300	57,400	800	99,000	1,400
	35	85,200	600	46,700	400	81,000	600
	35	64,900	500	35,300	200	61,000	400
	35	40,200	200	21,800	200	37,700	400
0.40	35	49,700	900	71,500	1,400	123,800	2,400
	35	40,000	600	57,500	1,000	99,500	1,700
	35	35,100	200	50,300	400	87,000	800
0.25	35	39,600	400	92,200	800	159,700	1,400
	35	35,000	300	81,100	1,100	140,500	1,900
0.15	37	32,000	500	123,200	1,200	213,400	2,000
	35	27,400	400	106,300	1,500	184,100	2,600
	35	26,000	300	100,300	1,200	173,700	2,000

* Rounded off to nearest 100 psi.

** $\overline{S}_m = \sqrt{3}\,\tau_{xym}$, using the von Mises-Hencky failure theory, and ${}^{\sigma}S_m = \sqrt{3}\,{}^{\sigma}\tau_{xym}$.

Table 23 Normal cycles-to-failure distribution parameters for Phase II results.

Stress Ratio, r_s	Alternating stress* psi	Cycles-to-failure		Skewness	Kurtosis	K-S Test Max. D Value****	Chi-squared	
		Mean	Standard Deviation***				Degrees of freedom	Value *****
∞	108,900	3,300	360	-0.163	2.317	0.094	1	0.248
	92,100	6,400	610	-0.325	1.672	0.134	2	6.151
	73,600	14,200	1,370	-0.447	2.654	0.108	2	0.880
	49,400	24,500	3,580	0.563	2.822	0.108	1	1.629
	39,300	86,200	16,460	0.784	3.180	0.116	1	4.900
1.06	105,700	4,500	650	0.252	2.786	0.109	1	1.265
	85,200	10,800	1,210	0.200	2.187	0.111	2	0.726
	64,900	24,300	2,760	0.757	3.095	0.139	1	1.095
	40,200	224,000	83,280	0.650	3.143	0.112	1	2.453
0.40	49,700	59,000	9,610	0.969	3.626	0.154	1	3.029
	40,000	210,700	36,740	0.735	3.364	0.103	1	0.631
	35,100	332,700	110,670	1.478	5.138	0.201	1	2.928
0.25	39,600	57,120	14,560	0.383	2.220	0.140	1	2.606
	35,000	95,700	22,490	0.597	2.790	0.099	2	3.106
0.15	32,100	148,000	31,800	0.526	2.758	0.093	1	1.017
	27,400	272,100	64,560	0.578	3.881	0.106	1	3.875
	26,000	504,100	156,480	1.560	6.049	0.194	0	Not applicable

* Rounded off to nearest 100 psi.

** Rounded off to nearest 100 cycles.

*** Standard deviation of cycles to failure rounded to nearest 10 cycles.

**** D critical at 0.05 level of significance and sample size of 35 is 0.224.

***** Accept/reject critical value for Chi-squared test at 0.05 level of significance:

Degrees of freedom	Critical value
1	3.841
2	5.991

Table 24 Lognormal cycles-to-failure distribution parameters for Phase II results.

Stress Ratio, r_s	Alternating stress* psi	Log_e Cycles-to-failure Mean**	Log_e Cycles-to-failure Standard Deviation**	Skewness	Kurtosis	K-S Test Max D Value***	Chi-squared Degrees of freedom	Chi-squared Value****
∞	108,900	8.095	0.112	-0.374	2.408	0.096	1	2.844
	92,100	8.760	0.097	-0.402	1.684	0.137	2	8.534
	73,600	9.554	0.100	-0.664	2.936	0.128	1	0.255
	49,400	10.097	0.143	0.246	2.568	0.097	1	2.608
	39,300	11.348	0.184	0.384	2.568	0.121	2	1.103
1.06	105,700	8.407	0.145	-0.141	2.991	0.087	1	0.288
	85,200	9.281	0.112	0.016	2.089	0.111	1	0.728
	64,900	10.093	0.110	0.513	2.751	0.117	2	1.595
	40,200	12.252	0.380	-0.175	2.347	0.114	2	8.188
0.40	49,700	10.973	0.155	0.567	3.126	0.122	0	Not applicable
	40,000	12.244	0.169	0.313	2.722	0.077	2	0.892
	35,100	12.670	0.296	0.682	3.237	0.142	2	1.206
0.25	39,600	10.921	0.257	-0.075	2.352	0.094	2	2.862
	35,000	11.442	0.231	0.145	2.291	0.068	2	0.528
0.15	32,100	11.883	0.212	0.101	2.300	0.095	2	4.410
	27,400	12.487	0.240	-0.237	3.278	0.086	1	3.011
	26,000	13.091	0.277	0.722	3.272	0.147	1	1.226

* Rounded off to nearest 100 psi.

** Rounded to three decimal places.

*** D critical at 0.05 level of significance and sample size of 35 is 0.224.

**** Accept/reject critical value for chi-squared test at 0.05 level of significance:

Degrees of freedom	Critical value
1	3.841
2	5.991

Table 25 Endurance strength calculations for Phase II grooved specimens at stress ratio of ∞.

Alternating Stress psi	i	n_i Successes	in_i	$i^2 n_i$
34,921	3	6	18	54
32,868	2	7	14	28
30,815	1	6	6	6
28,762	0	1	0	0
		N = 20	A = 38	B = 88

d = stress increment = 2,053 psi

X_o = lowest stress level = 28,762 psi

$\overline{X}$ = mean (estimate)

$\overline{X} = X_o + d[A/N + 1/2] = 28,762 + 2,053\left[\frac{38}{20} + \frac{1}{2}\right]$

$\overline{X}$ = 33,689 psi ≅ 33,700* psi

s = standard deviation (estimate)

$s = 1.620\, d[(NB-A^2)/N^2 + 0.029] = 1.620\,(2,053)\left[\frac{20(88) - (38)^2}{(20)^2} + 0.029\right]$

s = 2,724 psi ≅ 2,700* psi

* Rounded off to nearest 100 psi.

Table 26 Endurance strength calculations for Phase II grooved specimens at stress ratio of 1.06.

Alternating stress psi	i	n_i Failures	in_i	i^2n_i
35,202	5	1	5	25
34,109	4	3	12	48
33,016	3	8	24	72
31,923	2	3	6	12
30,830	1	4	4	4
29,737	0	1	0	0
		N = 20	A = 51	B = 161

d = stress increment = 1,093 psi average

X_o = lowest stress level = 29,737 psi

$\overline{X}$ = mean (estimate)

$\overline{X} = X_o + d[A/N - 1/2] = 29{,}737 + 1{,}093\ [\frac{51}{20} - 0.5]$

$\overline{X}$ = 31,978 psi ≅ 32,000 * psi

s = standard deviation (estimate)

$s = 1.620\ d[(NB-A^2)/N^2 + 0.029] = 1.620\ (1{,}093)\ [\frac{20(161) - (51)^2}{(20)^2} + 0.029]$

s = 2,796 psi ≅ 2,800 * psi

* Rounded off to nearest 100 psi.

Table 27 Endurance strength calculations for Phase II grooved specimens at stress ratio of 0.40.

Alternating stress psi	i	n_i Successes	in_i	i^2n_i
33,000	5	1	5	25
32,000	4	2	8	32
31,000	3	3	9	27
30,000	2	6	12	24
29,000	1	7	7	7
28,000	0	3	0	0
		N = 22	A = 41	B = 115

d = stress increment = 1,000 psi

X_o = lowest stress level = 28,000 psi

$\overline{X}$ = mean (estimate)

$\overline{X} = X_o + d[A/N + 1/2] = 28,000 + 1,000[\frac{41}{22} + \frac{1}{2}]$

$\overline{X}$ = 34,000 psi

s = standard deviation (estimate)

$s = 1.620\, d[(NB-A^2)/N^2 + 0.029] = 1.620\,(1,000)\left[\frac{22(115) - (41)^2}{(22)^2} + 0.029\right]$

s = 2,900* psi

* Rounded off to nearest 100 psi.

Table 28 Endurance strength calculations for Phase II grooved specimens at stress ratio of 0.25.

Alternating stress psi	i	n_i Failures	in_i	i^2n_i
27,000	4	1	4	16
26,000	3	4	12	36
25,000	2	6	12	24
24,000	1	8	8	8
23,000	0	3	0	0
		N = 22	A = 36	B = 84

d = stress increment = 1,000 psi

X_o = lowest stress level = 23,000 psi

$\overline{X}$ = mean (estimate)

$\overline{X} = X_o + d[A/N - 1/2] = 23,000 + 1,000\left[\frac{36}{22} - \frac{1}{2}\right]$

$\overline{X}$ = 24,136 psi ≅ 24,100 * psi

s = standard deviation (estimate)

$s = 1.620\, d[(NB-A^2)/N^2 + 0.029] = 1,620\ (1,000)\left[\frac{22(84) - (36)^2}{(22)^2} + 0.029\right]$

s = 1,732 psi ≅ 1,700* psi

* Rounded off to nearest 100 psi.

Table 29 Endurance strength calculations for Phase II grooved specimens at stress ratio of 0.15.

Alternating stress psi	i	n_i Failures	in_i	i^2n_i
25,000	3	1	3	9
24,000	2	8	16	32
23,000	1	12	12	12
22,000	0	1	0	0
		N = 22	A = 31	B = 53

d = stress increment = 1,000 psi

X_o = lowest stress level = 22,000 psi

$\bar{X}$ = mean (estimate)

$\bar{X} = X_o + d[A/N - 1/2] = 22,000 + 1,000 \left[\frac{31}{22} - \frac{1}{2} \right]$

$\bar{X}$ = 22,900* psi

s = standard deviation (estimate)

$s = 1.620\, d[(NB-A^2)/N^2 + 0.029] = 1.620\ (1,000) \left[\frac{22(53) - (31)^2}{(22)^2} + 0.029 \right]$

s = 700* psi

* Rounded off to nearest 100 psi.

Table 30 Summary of Phase II endurance strength results at 2.5×10^6 cycles of life for AISI 4340 steel R_c 35/40 grooved specimens for distributional Goodman diagram.

Stress ratio, r_s	Nominal alternating bending stress, S_a		Nominal normal mean stress, S_m		Nominal combined stress vector**, S_v	
	Mean*, $\bar{S}_a$ psi	Std. dev.* S_a psi	Mean*, $\bar{S}_m$ psi	Std. dev.* σ_{S_m} psi	Mean*, $\bar{S}_v$ psi	Std. dev.* σ_{S_v} psi
∞	33,700	2,700	0	0	33,700	2,700
1.06	32,000	2,800	30,200	2,600	44,000	3,800
0.40	30,400	2,900	76,000	7,200	81,900	7,800
0.25	24,100	1,700	96,500	6,900	99,500	7,100
0.15	22,900	700	152,700	4,700	154,400	4,700

* Rounded off to nearest 100 psi.

** $\bar{S}_m = \frac{\bar{S}_a}{\bar{r}_s}$ and $\sigma_{S_m} = \frac{1}{\bar{r}_s} \sigma_{S_a}$.

*** $\bar{S}_v = \bar{S}_a \left(1 + \frac{1}{\bar{r}_s^{\,2}}\right)^{1/2}$ and $\sigma_{S_v} = \sigma_{S_a} \left(1 + \frac{1}{\bar{r}_s^{\,2}}\right)^{1/2}$.

Table 31 K-S and Chi-squared goodness-of-fit test results for seventeen sets of Phase II cycles-to-failure data fitted to the Weibull distribution.

Stress ratio, r_s	Nominal alternating bending stress Mean psi	Sample size	Weibull distribution parameters			K-S test Max. D* Value	Chi-squared test Variable cell	
			β	η cycles	γ cycles		Degrees of freedom	Value**
∞	108,900	35	3.85	1,436	2,000	0.086	2	0.489
	92,100	35	1.09	1,160	5,400	0.194	2	26.317
	73,600	35	8.38	10,653	4,100	0.074	3	4.436
	49,400	35	2.24	8,625	16,900	0.082	2	2.826
	39,300	35	1.75	31,880	58,100	0.099	3	12.240
1.06	105,700	35	4.15	2,674	2,100	0.122	2	10.062
	85,200	35	2.02	2,849	8,300	0.083	3	6.527
	64,900	35	1.84	5,589	19,400	0.100	3	2.300
	40,200	35	1.47	153,893	89,000	0.110	2	5.231
0.40	49,700	35	2.04	20,518	40,900	0.124	3	3.109
	40,000	35	2.02	80,214	140,000	0.070	2	2.290
	35,100	35	1.62	177,305	174,000	0.135	3	3.169
0.25	39,600	35	2.35	36,851	24,600	0.110	2	5.420
	35,000	35	1.53	41,500	59,000	0.080	2	6.401
0.15	32,100	35	1.65	62,546	93,000	0.079	2	2.209
	27,400	37	2.98	198,228	95,500	0.108	2	3.365
	26,000	35	1.41	225,960	299,300	0.115	3	9.121

* K-S D value = 0.224.

** Chi-squared value, 2 degrees of freedom = 5.991.

** Chi-squared value, 3 degrees of freedom = 7.815.

These critical goodness-of-fit test values are for 0.05 level of significance.

APPENDICES

APPENDIX A

PDP PROGRAM TO CALCULATE VISICORDER DIVISIONS FOR DESIRED LEVELS OF BENDING AND TORQUE

```
C-8K MODV 11-219

01.09 T "MACHINE NO. 3", !, !, !
01.10 T "STRESS RATIO = INFINITY", !
01.20 A "STRESS LEVEL = ", SL, !
01.30 A "BENDING CAL. RESIST. =", RB, !
01.50 S K1 = 1.3081
01.60 S K2 =  .0170
01.70 S K3 =  .8748
01.80 S K4 =  .0542
01.90 S K5 =  .0645

02.06 S DS =   .491
02.07 S DO = 2.0
02.08 S DI = 1.3125
02.20 S NB = 50
02.30 S NA =  4
02.40 S GB = 3.23
02.50 S  A = 30000000
02.60 S BR = 190
02.70 S NT =  45
02.80 S GT = 2.06
02.90 S TR + 120

03.10 S TO = 0
03.20 S SO = K2*K1*SL
03.30 S SP = SO+K4*TO
03.40 S VB = (NB*NA*GB*RB*SP)/(A*BR)
03.80 T %7.03, "VIS. DIV. - BENDING", VB, !
03.90 T %8.01, "BENDING CALIB.", RB," FOR 50 DIV.,", !

05.10 S EB = .03*VB
05.20 S ET + .03*VT
05.30 S SB + SL/VB
05.50 T %5.02 "DIV. OF BENDING FOR 3% ERROR", EB, !
05.70 T %8.01 "BENDING SENSITIVITY", SB," PSI/DIV.", !
```

APPENDIX B

PROGRAM STRESS (Updated)

The purpose of this program is to convert each Visicorder record into normal stress, shear stress and stress ratio. The program calculates the mean and standard deviation of these two stresses and of the stress ratio of each group of cycles-to-failure data. It calculates the cycles-to-failure from time-to-failure data. The program will also accept cycles-to-failure data.

This program remains basically as described previously. Added are the latest calibration coefficients and a routine to convert shear stresses to normal mean stresses.

The program distinguishes between cycles-to-failure and time-to-failure through the use of a code number. The code also tells the program whether or not the group of data are endurance test data. This discrimination is necessary because the program will not calculate the endurance strength distribution parameters. The discrimination code is fed in as data, and is as follows:

0 if the failure data is in time to failure.

1 if the data is endurance test data.

2 if the failure data is in cycles to failure.

The program will accept as many sets of data as desired and the groups may be mixed; i.e., endurance test, group with cycles-to-failure data, and group with time to failure data. A group is all the data for one stress level. A group of data consists of the following: The first card contains in this order, the number of specimens in the level, the mode of the run and the code. The mode is dependent upon the date the run was made and identifies the calibration constants for each machine to be used in the computations.

The fields on the data card are as follows:

spaces 1 to 5 - number of specimens

spaces 6 to 10 - mode

spaces 11 to 15 - code.

The number of specimens, mode and code are fixed point numbers and have no decimals but the numbers must be placed to the right in each field.

The next sequence of cards reads in the cycles-to-failure or time to failure in hours, minutes and seconds. If the data is in time to failure there are ten groups on a card, so the number of cards required will depend upon how many specimens are in the stress level. The format across the card is as follows:

spaces 1 and 2 - blank
spaces 3 and 4 - hours
spaces 5 and 6 - minutes
spaces 7 and 8 - seconds
spaces 9 and 10 - blank.

and the sequence continues in this manner. If the failure data is in cycles-to-failure the format is 8 fields of 10 spaces each and the decimals appear in the last space of each field; i.e. spaces 10, 20, 30, etc. If the group of data is for an endurance test there is no failure data and these cards are left out. The program will automatically handle the data if the proper code number is put on the first card.

Following the cycles-to-failure cards are the cards containing the information for each specimen in the stress level. The information must be placed on each card as follows:

spaces 1 to 5 - test number

spaces 6 to 10 - specimen number

spaces 11 to 15 - machine number

spaces 16 to 20 - with a decimal in space 20 - pan weight

spaces 21 to 30 with a decimal in space 28 - bending calibration resistance

spaces 31 to 40 with a decimal in space 38 - number of bending calibration divisions

spaces 41 to 50 with a decimal in space 48 - number of divisions of bending

spaces 51 to 60 with a decimal in space 58 - torque calibration resistance

spaces 61 to 70 with a decimal in space 68 - number of torque calibration divisions

spaces 71 to 80 with a
decimal in space 78 - number of divisions of torque

The test number, specimen number, and machine number are fixed point numbers and must be placed to the right in each field. There is one card for each test specimen and the cards must be placed in the same order as the failure data is placed on the cards preceeding these cards. For data at stress ratio of ∞ there will be no torque stress data. In this case these fields can be left blank. The computer reads blanks on data cards as zeros.

This makes up one group of data at a given stress level and ratio. As many groups may be run as desired by simply placing the groups one behind the other.

A list of important variables and symbols in the program STRESS using Fortran language follows:

List of Definitions for Program to Find
Stress Levels and Ratios (PROGRAM STRESS)

NCARDS	=	number of specimens tested at given level.
MODE	=	number of mode depending on date of test.
NCODE	=	0 if failure data is in times to failure.
	=	1 if data is from an endurance level.
	=	2 if data is in cycles-to-failure.
XHOURS(I) XMIN(I) SECS(I)	=	times to failure in hours, minutes and seconds.
TOTCY(I)	=	cycles-to-failure.
NOTEST	=	test number.
NOSPEC	=	specimen number.
MACHNO	=	machine number.
PANWT	=	amount of weight on loading arm.
RCALB	=	calibration resistance used in bending channel.
ENCLAB	=	number of visicorder divisions used when calibrating bending channel.
ENVISB	=	number of divisions during actual test.
RCALT	=	calibration resistance used in torque channel.
ENCALT	=	number of visicorder divisions used when calibrating torque channel.
ENVIST	=	number of divisions during actual test.
ENA	=	number of active arms in strain gage bridge.

RGAGEB = resistance of bending strain gage.

RGAGET = resistance of torque strain gages.

GB = bending gage factor.

GT = torque gage factor.

CBGR = calibration constant K_{BGR}.

CGRTH = calibration constant K_{GR-TH}.

CT = calibration constant K_T.

CTB = calibration constant $K_{T/B}$.

RPM = revolutions per minute of machine.

SOUTH = output normal stress corrected for interaction.

TAUTH = output shear stress corrected for interaction.

STRGR(I) = normal stress in specimen groove.

TAUGER(I) = shear stress in specimen groove.

SOUTHP = output stress not corrected for interaction.

TAUTHP = output stress not corrected for interaction.

The program STRESS listing in Fortran language follows:

```
      PROGRAM STRESS (INPUT,OUTPUT,TAPE1 =INPUT)
      DIMENSION STRGR(50), TAUGR(50), R(50),XHOURS(50),XMIN(50),
     1SECS(50),TOTCY(50)
C     READ IN THE NUMBER OF CARDS IN THE STRESS LEVEL AND THE MODE
C     OF OPERATION OF THE MACHINE
C-----NCODE = 1 FOR ENDURANCE LEVEL, NCODE = 0 IF FAILURES IN TIMES TO FAILURE.
C--------NCODE = 2 IF FAILURES IN CYCLES TO FAILURE.
35    READ 100, NCARDS, MODE, NCODE
100   FORMAT (3I5)
      IF (EOF,1) 201, 171
171   PRINT 91
91    FORMAT(1H1//)
170   IF (NCODE.EQ.1) GO TO 175
C-----FORMAT FOR OUTPUT HEADINGS INCLUDING TIMES TO FAILURE.
      PRINT 310, MODE
  310 FORMAT(62X,11HTEST MODE =,I2/)
      PRINT 61
61    FORMAT (2X,34HTEST  SPEC.  MACH.  PAN     CYCLES,7X,4HRCAL,7X
     1,4HNCAL,7X,4HNVIS,6X,4HRCAL,6X,4HNCAL,6X,4HNVIS,5X,7HBENDING,4X, 1
     26HSHEAR     STRESS/3X,35HNO.    NO.    NO.    WT.    TO FAILURE,3X,
     37HBENDING,4X,7HBENDING,4X,7HBENDING,4X,6HTORQUE,4X,6HTORQUE,4X,
     46HTORQUE,5X,6HSTRESS,4X,6HSTRESS,4X,5HRATIO//)
      IF (NCODE.EQ.2) GO TO 320
C------ROUTINE TO READ IN TIMES TO FAILURE.
      READ 401, (XHOURS(I),XMIN(I),SECS(I), I = 1, NCARDS)
 401  FORMAT (10(2X,3F2.0))
      GO TO 300
C-----READ IN CYCLES TO FAILURE.
320   READ 402, (TOTCY(I), I=1,NCARDS)
402   FORMAT ( 8F10.0)
      GO TO 300
C-----FORMAT FOR OUTPUT HEADINGS WITHOUT TIMES TO FAILURE.
175   PRINT 90, MODE
90    FORMAT (50X,14HENDURANCE TEST,10X,11HTEST MODE =,I2/)
172   PRINT 60
   60 FORMAT (3X,4HTEST,3X,8HSPECIMEN,3X,7HMACHINE,5X,3HPAN,7X,4HRCAL,7X
     1,4HNCAL,7X,4HNVIS,6X,4HRCAL,6X,4HNCAL,6X,4HNVIS,5X,7HBENDING,4X,
     25HSHEAR,4X,6HSTRESS/4X,3HNO.,6X,3HNO.,7X,3HNO.,5X,6HWEIGHT,4X,
     37HBENDING,4X,7HBENDING,4X,7HBENDING,4X,6HTORQUE,4X,6HTORQUE,4X,
     46HTORQUE,5X,6HSTRESS,4X,6HSTRESS,4X,5HRATIO/ )
C-----ROUTINE TO CALCULATE POLAR MOMENTS OF INERTIA.
C-----SPDIA = SPECIMEN DIA.    TOD = TOOLHOLDER O. D.   TID = TOOLHOLDER I. D.
  300 SPDIA=.4910
      TOD=2.0
      TID=1.3125
      SPJC=(3.14159*SPDIA**3)/16.0
      THJC = (3.14159*(TOD**4-TID**4))/(16.*TOD)
      CONST=THJC/SPJC
      DO 120 I = 1, NCARDS
C     READ IN THE TEST NO., SPECIMEN NO., MACHINE NO., PANWEIGHT, AND THE
```

```
C     CALIBRATION RESISTANCE, VISICORDER CALIBRATION DISTANCE AND VISICORDER
C     OUTPUT DIVISIONS FOR BENDING AND TORQUE
   80 READ 10,NOTEST,NOSPEC,MACHNO, PANWT,RCALB,ENCALB,ENVISB,RCALT,
     1ENCALT,ENVIST
10    FORMAT (3I5,F5.0,6F10.2)
      CARDS = NCARDS
C     DEFINE THE ELASTIC MODULUS, NO. OF ACTIVE ARMS OF THE BRIDGES, THE
C     RESISTANCES OF THE BENDING AND TORQUE GAUGES, AND THE BENDING AND
C     TORQUE GAUGE FACTORS
      E=30000000.
      ENA=4.
      RGAGEB=190.
      RGAGET=120.
      GB=3.23
      GT=2.06
C     SELECTION OF MACHINE AND MODE
      IF(MACHNO.EQ.1.AND.MODE.EQ.1) GO TO 11
      IF(MACHNO.EQ.2.AND.MODE.EQ.1) GO TO 24
      IF(MACHNO.EQ.3.AND.MODE.EQ.1) GO TO 31
      IF(MACHNO.EQ.1.AND.MODE.EQ.2) GO TO 11
      IF(MACHNO.EQ.2.AND.MODE.EQ.2) GO TO 24
      IF(MACHNO.EQ.3.AND.MODE.EQ.2) GO TO 31
      IF(MACHNO.EQ.1.AND.MODE.EQ.3) GO TO 11
      IF(MACHNO.EQ.2.AND.MODE.EQ.3) GO TO 24
      IF(MACHNO.EQ.3.AND.MODE.EQ.3) GO TO 33
      IF(MACHNO.EQ.1.AND.MODE.EQ.4) GO TO 11
      IF(MACHNO.EQ.2.AND.MODE.EQ.4) GO TO 24
      IF(MACHNO.EQ.3.AND.MODE.EQ.4) GO TO 34
      IF(MACHNO .EQ. 1 .AND. MODE .EQ. 5) GO TO 15
      IF(MACHNO .EQ. 2 .AND. MODE .EQ. 5) GO TO 25
      IF(MACHNO .EQ. 3 .AND. MODE .EQ. 5) GO TO 53
C     CALIBRATION PARAMETERS FOR GIVEN MODE AND MACHINE
11    CBGR = 1.0123
      CGRTH = .0208
      CT = .8752
      CTB = -.0459
      CBT = .029
      RPM=1786.
      GO TO 50
   15 CBGR=1.3081
      CGRTH=0.0149
      CT=0.8753
      CTB=-0.0462
      CBT=0.0493
      RPM = 1779
      GO TO 50
24    CBGR = 1.0123
      CGRTH = .0188
```

```
      CT = .8201
      CTB = 0.0344
      CBT = .0422
      RPM=1784.
      GO TO 50
   25 CBGR=1.3081
      CGRTH=0.0145
      CT=0.8950
      CTB=0.0463
      CBT=0.0544
      RPM = 1775
      GO TO 50
31    CBGR = 1.0946
      CGRTH = .0211
      CT = .933
      CTB = .0
      CBT = -.0149
      RPM=1780.
      GO TO 50
33    CBGR = 1.0946
      CGRTH = .0211
      CT = .7721
      CTB = .0
      CBT = -.0127
      RPM=1780.
      GO TO 50
34    CBGR = 1.0123
      CGRTH = .0197
      CT = .7721
      CTB = .0
      CBT = -.0127
      RPM=1780.
   53 CBGR=1.3081
      CGRTH=0.0170
      CT=0.8748
      CTB=0.0542
      CBT=0.0645
      RPM = 1778
      GO TO 50
   50 IF(ENVIST.EQ.0.0) GO TO 160
      GO TO 51
C     CALCULATION OF BENDING STRESS LEVEL FOR INFINITY RATIO
  160 SOUTH=(ENVISB*E*RGAGEB)/(ENCALB*ENA*GB*RCALB)
      STRGR(I) = SOUTH / (CGRTH * CBGR)
      TAUGR(I) = 0.0
      IF (NCODE.EQ.1) GO TO 301
      IF (NCODE.EQ.2) GO TO 330
C-----CALCULATE CYCLES TO FAILURE FROM TIMES TO FAILURE.
      CYHR = XHOURS(I)*60.*RPM
      CYMIN = XMIN(I)*RPM
```

```
      CYSEC = (SECS(I)*RPM)/60.0
      TCY = CYHR+CYMIN+CYSEC
      GO TO 335
330   TCY = TOTCY(I)
335   PRINT 72,NOTEST,NOSPEC,MACHNO,PANWT,TCY,RCALB,ENCALB,ENVISB,RCALT,
     1ENCALT,ENVIST,STRGR(I),TAUGR(I)
72    FORMAT(3I6,F8.1,2F11.0,F10.2,F11.2,F11.0,F10.2,
     1F10.2,F11.0,F10.0,4X,6HINFIN./)
      GO TO 120
301   PRINT 71,NOTEST,NOSPEC,MACHNO, PANWT,RCALB,ENCALB,ENVISB,RCALT,
     1ENCALT,ENVIST,STRGR(I),TAUGR(I)
71    FORMAT(4X,I3,6X,I3,8X,I1,6X,F5.1,F12.0,F10.2,F11.2,F11.0,F10.2,
     1F10.2,F11.0,F10.0,3X,6HINFIN./)
      GO TO 120
C     CALCULATION OF BENDING STRESS, SHEAR STRESS AND STRESS RATIO FOR
C     ALL FINITE RATIOS
   51 SOUTHP=(ENVISB*E*RGAGEB)/(ENCALB*ENA*GB*RCALB)
      TAUTHP=(ENVIST*E*RGAGET)/(ENCALT*ENA*GT*RCALT)
      SOUTH=SOUTHP-CTB*TAUTHP
      TAUTH=TAUTHP-CBT*SOUTH
      STRGR(I) = SOUTH / (CGPTH * CBGR)
      TAUGR(I) = CT*TAUTH*CONST
      R(I) = STRGR(I) / (TAUGR(I) * 1.732)
      IF (NCODE.EQ.1) GO TO 302
      IF (NCODE.EQ.2) GO TO 340
C-----CALCULATE CYCLES TO FAILURE FROM TIMES TO FAILURE.
      CYHR = XHOURS(I)*60.*RPM
      CYMIN = XMIN(I)*RPM
      CYSEC = (SECS(I)*RPM)/60.0
      TCY = CYHR+CYMIN+CYSEC
      GO TO 345
340   TCY = TOTCY(I)
345   PRINT 73,NOTEST,NOSPEC,MACHNO,PANWT,TCY,RCALB,ENCALB,ENVISB,RCALT,
     1ENCALT,ENVIST,STRGR(I),TAUGR(I),R(I)
73    FORMAT(3I6,F8.1,2F11.0,F10.2,F11.2,F11.0,F10.2,
     1F10.2,F11.0,F10.0,F9.3/)
      GO TO 120
302   PRINT 70,NOTEST,NOSPEC,MACHNO, PANWT,RCALB,ENCALB,ENVISB,RCALT,
     1ENCALT,ENVIST,STRGR(I),TAUGR(I),R(I)
70    FORMAT(4X,I3,6X,I3,8X,I1,6X,F5.1,F12.0,F10.2,F11.2,F11.0,F10.2,
     1F10.2,F11.0,F10.0,F9.3/)
  120 CONTINUE
      IF (NCODE.EQ.1.AND.ENVIST.EQ.0.0) GO TO 35
      IF (NCODE.EQ.1) GO TO 200
C     CALCULATION OF MEAN AND STANDARD DEVIATION OF BENDING STRESS, SHEAR
C     STRESS, AND STRESS RATIO
      CALL MEAN (STRGR,  CARDS,NCARDS, XMEAN, DEV)
      PRINT 3
3     FORMAT (1H )
      PRINT 130, XMEAN, DEV
```

```
130   FORMAT (19X,31HMEAN BENDING STRESS IN GROOVE =,F10.1,5H PSI.//11X,
     139HSTD. DEV. OF BENDING STRESS IN GROOVE =,F10.2,5H PSI./)
132   IF(ENVIST.EQ.0.0) GO TO 35
      IF (NCODE.EQ.1) GO TO 200
      CALL MEAN (TAUGR, CARDS, NCARDS, XMEAN, DEV)
      PRINT 3
      PRINT 140, XMEAN, DEV
140   FORMAT (20X,30HMEAN TORQUE STRESS IN GROOVE =,F10.1,5H PSI./ 12X,
     138HSTD. DEV. OF TORQUE STRESS IN GROOVE =,F10.2,5H PSI. )
      SMEAN = XMEAN*1.732
      SDEV = DEV*1.732
      PRINT 3
      PRINT 142, SMEAN, SDEV
 142  FORMAT(19X, *MEAN NORMAL STRESSSIN GROOVE*, F10.1, * PSI.*//11X,
     1*STD. DEV. OF NORMAL MEAN STRESS IN GR =*, F10.2, * PSI.*/)
200   CALL MEAN (     R,  CARDS,NCARDS, XMEAN, DEV)
      PRINT 3
      PRINT 150, XMEAN, DEV
150   FORMAT (31X,19HMEAN STRESS RATIO =,F10.5/ 23X,27HSTD. DEV. OF STRE
     1SS RATIO =,F10.5)
37    GO TO 35
201   STOP
      END
      SUBROUTINE MEAN (X, DATA, NDATA, XMEAN, DEV)
C-----SUBROUTINE TO CALCULATE THE MEAN AND STANDARD DEVIATION OF DATA.
      DIMENSION X(NDATA)
      SIGMA= 0.0
      DO 8 I=1, NDATA
8     SIGMA=SIGMA+ X(I)
      XMEAN = SIGMA/DATA
      TOP2 = 0.0
      DO 9 I=1,NDATA
9     TOP2 = TOP2 + (X(I) - XMEAN)**2
      DEV =SQRT(TOP2/(DATA - 1.0))
      RETURN
      END
```

APPENDIX C

PROGRAM CYTOFR (Updated)

This program calculates estimates of the mean and standard deviation of the cycles-to-failure data for both the normal and the lognormal distributions, and calculates the moment coefficients of skewness and kurtosis. It also performs the Chi-squared and the Kolmogorov-Smirnov goodness-of-fit tests.

The program has been updated since it was previously reported.

The main program was revised to incorporate a sort routine to preclude the necessity for manually ordering the data inputs. Subroutine CHISQA (Chi-squared test) was modified to provide for automatic combining of cells at the tails of the distribution when the end cells do not contain at least five failure data points. Subroutine GRAPH was added to the program to plot a histogram of the cycles-to-failure data based on the cell widths and number of failure data points per cell computed by the CHISQA subroutine. The theoretical distribution curve represented by the parameters estimated by the main program is sketched and superimposed over the histogram. The plotting of the histogram and of the distribution is done by the Cal-Comp plotter from an output tape generated by the computer.

The data deck for operating program CYTOFR is in three logical sections per problem. The first section consists of two cards. Card one consists of three titles, while card two specifies the parameters necessary for the statistical calculations in CYTOFR.

The second, logical section is a variable number of cards, each specifying up to eight data points for analysis.

Section three provides parameters for plotting the normal and lognormal distributions. The first card is a parameter list for the normal distribution plot, followed by exactly six cards of footnotes. Next is a parameter list card for the lognormal distribution.

Multiple problems may be executed by stacking complete data sets behind each other. A list of important variables and symbols in program CYTOFR using Fortran language follows:

List of Definitions for Program to fit Normal and Log-Normal Distributions to Cycles-to-Failure Data (PROGRAM CYTOFR)

Main Program:

NDATA = DATA = number of observations.

STRLV = stress level in psi.

AKURCY = accuracy to which cycles-to-failure data are known.

RATIO = stress ratio

X(I) = cycles-to-failure data

CUMFRQ(I) = cumulative frequency of each X(I); ie, number of X's less than or equal to X(I).

PCAREA(I) = CUMFRQ(I)/NDATA

Subroutine to calculate the mean and standard deviation of the cycles-to-failure data (SUBROUTINE MEAN)

SIGMA = sum of the X(I)'s

XMEAN = average of the X(I)'s

TOP2 $= \sum_{i=i}^{\eta} (X(I)-XMEAN)^2$

DEV = standard deviation of the X(I)'s

Function subroutine to find the area under the normal curve (FUNCTION PROB(X)).

X = abscissa value for which corresponding area is desired.

PROB = desired area.

Subroutine for Chi-square goodness-of-fit test (SUB-ROUTINE CHISQA).

K = number cells.

XMAX = largest value of cycles-to-failure.

XMIN = smallest value of cycles-to-failure.

CSV = cell starting value.

CEV = cell end value.

CLB = cell lower bound.

CUB = cell upper bound.

FREQ(J) = number of observations in J^{th} cell.

REQAREA(J) = expected value of J^{th} cell.

CHISQR = total Chi-square value.

U(I) = Chi-square value of I^{th} cell.

Subroutine for Kolmogorov-Smirnov test (SUBROUTINE DTEST).

Z(I) = abscissa value on standard normal curve for a given X(I).

ARUNCN = area under standard normal curve from - to Z(I).

DSTAT(I) = absolute difference between the data cumulative frequency and the hypothesized cumulative frequency.

XMEAN = average of the X(I)'s.

DEV = standard deviation of the X(I)'s

PROB(T) = area under the standard normal curve from -T to +T.

Subroutine to calculate the moment coefficients of skewness and kurtosis (SUBROUTINE ALPHA).

ALPHA3 = moment coefficient of skewness.

ALPHA4 = moment coefficient of skewness.

VAR $= \sum_{i=i}^{n} (X(I)-\underline{X})^2$

TOP3 $= \sum_{i=i}^{n} (X(I)-\underline{X})^3$

SKEW = third moment of the data.

STDEV = biased estimator for standard deviation.

TOP4 $= \sum_{i=i}^{n} (X(I)-\underline{X})^4$

TKURT = fourth moment of the data.

DATA DECK STRUCTURE

Card	Columns	Description
1	1-20	Twenty character descriptive title to appear at the top of each printed output page.
	21-40	Twenty character descriptive title of the input data will appear on printed output, as well as the X-axis label for both slots.
	41-50	Unit of data measurement (cycles, inches, etc.).
2	1-10	Number of data points on following card(s). Must have a decimal point.
	11-20	Stress level. Must have a decimal point.
	21-30	Stress ratio. Must have a decimal point.
	31-40	Accuracy. Must have a decimal point.
3 to n**	1-10, 11-20, ..., 71-80	Data points, punched eight per card until all points are exhausted. Must have decimal points.
n + 1	1-10	The letters "NORMAL" followed by four blank spaces.
	11-20	Length of the X-axis in inches. If zero or blank, 6.0 is assumed. Decimal point is necessary.
	21-30	Length of the X-axis in inches. If zero or blank, 5.0 is assumed; if greater than 5.0, 5.0 is assumed. Decimal point is necessary.

**The notation 3rd to n is intended to mean from the third card to the nth card. In the case of 34 data points, the second section of data would stretch from the 3rd to 7th card .

DATA DECK STRUCTURE (Continued)

Card	Columns	Description
	31-40	Minimum X-axis value. The plotting program may find it necessary to alter this value slightly. If absent or zero, a resonable maximum will be assumed. The decimal point is necessary.
	41-50	Minimum Y-axis value. This should be blank or 0.
	51-60	Maximum X-axis value. Follow same rules as minimum X-axis value.
	61-70	Maximum Y-axis value. If absent or zero, the plotting program will search for the smallest even number greater than (or equal to) the height of the tallest histogram block, an automatic adjustment will be made. The decimal point is necessary.
	71-80	Height of lettering on graph, if 0 < height < .15. Otherwise this parameter will be set to equal .15.
n + 2 to n + 7	1-50	Footnotes punched as they will appear on the normal graph.
n + 8	1-10	The letters "LOG-NORMAL." Otherwise the same as card n + 1.

The program for CYTOFR listing in Fortran language follows:

```
      PROGRAM CYTOFR (INPUT,OUTPUT,TAPE1=INPUT,TAPE2)
C-----PROGRAM TO FIT NORMAL AND LOG-NORMAL CURVE TO DATA AND CHECK
C-----GOODNESS OF FIT.
      COMMON CUMFRQ(100),NDATA,X(100),DEV,XMEAN,CLB(9),CUB(9),FREQ(9),K,
     1 CHISQR,TITLE(2),SUBTITL(2),CSV(9),CEV(9),PCAREA(100),DSTAT(100),
     2  AREA(9),REQAREA(9),EXFREQ(9),U(9),Z(100),NX(100),DATA,AKURCY,
     3 XMAX,XMIN,PSI,CD,D,RATIO,XLENGTH,YLENGTH,YMAX,YMIN,XMA,XMI,
     4 HLETTER,COM(3),W,ALPHA3,ALPHA4,FOOT(30),IT,SKS,UNIT
      INTEGER TITLE,SUBTITL,FOOT
      EXTERNAL PROB
C
C     INITIALIZE PLOTTER
C
      CALL INITIAL (0,2,0.3,0)
710   PRINT 1
C-----NDATA=DATA=NUMBER OF OBSERVATIONS
C-----STRLV = STRESS LEVEL IN PSI.
C     X=NUMBER OF CYCLES TO FAILURE
      READ 500,TITLE,SUBTITL,UNIT
  500 FORMAT(8A10)
      IF(EOF(1)) 56,5
    5 READ 501,DATA,STRLEV,RATIO,AKURCY
  501 FORMAT(8F10.0)
      NDATA=DATA
      READ 501,(X(I),I=1,NDATA)
C
C     SORT X(I) TERMS IN ASCENDING ORDER.
C
      K=NDATA-1
      IF(K.LE.0) GO TO 30
      DO 20 I=1,K
      N=NDATA-I
      ISTOP=0
      DO 10 J=1,N
      IF(X(J).LE.X(J+1)) GO TO 10
      SAVE=X(J)
      X(J)=X(J+1)
      X(J+1)=SAVE
      ISTOP=ISTOP+1
   10 CONTINUE
      IF(ISTOP.EQ.0) GO TO 30
   20 CONTINUE
C
C     SET CUMFRQ(I) ARRAY
C
   30 DO 40 I=1,NDATA
   40 CUMFRQ(I)=I
C
C     RESET SOME CUMFRQ(I) ENTRYS IF X(I)=X(I+1) OCCURS
```

```
C
      DO 50 I=2,NDATA
   50 CONTINUE
C-----PCAREA = F(N) OF OBSERVATIONS
      DO 759 I=1, NDATA
759   PCAREA(I) = CUMFRQ(I)/DATA
      PRINT 408,TITLE,SUBTITL
  408 FORMAT(*1*,55X,2A10,/,42X,*NORMAL DISTRIBUTION FITTED TO *,2A10,
     1///)
      IF (RATIO.EQ.0.0) GO TO 414
      PRINT 402, STRLEV, RATIO
  402 FORMAT(29X,*STRESS LEVEL =*,F7.1,* PSI.*,16X
     1,14HSTRESS RATIO =,F6.3//)
      GO TO 415
414   PRINT 416, STRLEV


  416 FORMAT(29X,*STRESS LEVEL =*,F7.1,* PSI.*,16X
     1,23HSTRESS RATIO = INFINITY//)
  415 PRINT 404,SUBTITL
  404 FORMAT(56X,2A10,/)
      PRINT 403, (X(I),I=1,NDATA)
  403 FORMAT(6F21.4)
      PRINT 3
3     FORMAT (1H0)
      CALL MEAN
      CALL CHISQA
      CALL DTEST
      CALL ALPHA
      READ 502,IT,XLENGTH,YLENGTH,XMI,YMIN,XMA,YMAX,HLETTER
  502 FORMAT(A1,9X,7F10.0)
      READ 503,FOOT
  503 FORMAT(5A10)
      IF(IT.NE.1HN) GO TO 57
      CALL GRAPH
   53 AKURCY=.00001
      DO 54 I=1,NDATA
      NX(I) =(ALOG(X(I)/20.)+ALOG(20.))*100000.  + .5
      X(I) = NX(I)
54    X(I) = X(I)/100000.
      PRINT 1, TITLE
    1 FORMAT(*1*,55X,2A10)
      PRINT 401,SUBTITL
  401 FORMAT(38X,*LOG-NORMAL DISTRIBUTION FITTED TO *,2A10,///)
      IF (RATIO.EQ.0.0) GO TO 417
      PRINT 402, STRLEV, RATIO
      GO TO 418
417   PRINT 416, STRLEV
  418 PRINT 2,SUBTITL
    2 FORMAT(49X,*LOGS OF THE *,4A10,/)
      PRINT 413, (X(I),I=1,NDATA)
```

```
  413 FORMAT(6(8X,F12.4))
      PRINT 3
      CALL MEAN
      CALL CHISQA
      CALL DTEST
      CALL ALFHA
      READ 502,IT,XLENGTH,YLENGTH,XMI,YMIN,XMA,YMAX
      IF(IT.NE.1HL) GO TO 57
      CALL GRAPH
      GO TO 710
   56 CALL PLOT (0.,0.,999)
      CALL EXIT
   57 CALL PLOT (0.,0.,999)
      STOP 1111
      END
      SUBROUTINE MEAN
C-----SUBROUTINE TO CALCULATE THE MEAN AND STANDARD DEVIATION OF DATA.
      COMMON CUMFRQ(100),NDATA,X(100),DEV,XMEAN,CLB(9),CUB(9),FREQ(9),K,
     1 CHISQR,TITLE(2),SUBTITL(2),CSV(9),CEV(9),PCAREA(100),DSTAT(100),
     2  AREA(9),REQAREA(9),EXFREQ(9),U(9),Z(100),NX(100),DATA,AKURCY,
     3  XMAX,XMIN,PSI,CD,D,R,XLENGTH,YLENGTH
      SIGMA= 0.0
      DO 8 I=1, NDATA
8     SIGMA=SIGMA+ X(I)
      XMEAN = SIGMA/DATA
      TOP2 = 0.0
      DO 9 I=1,NDATA
9     TOP2 = TOP2 + (X(I) - XMEAN)**2
      DEV =SQRT(TOP2/(DATA - 1.0))
      PRINT 14, XMEAN


      PRINT 15, DEV
   14 FORMAT(10X,*SAMPLE MEAN =*,F14.4)
   15 FORMAT(10X,*STD. DEVIATION=*,F12.4)
      RETURN
      END
      FUNCTION PROB(X)
C-----THIS SUBROUTINE GIVES AREA UNDER NORMAL CURVE FROM -Z TO +Z
C     WITH AN ACCURACY OF 0.00005
C-----Z VALUE GIVEN BY CALLING PROGRAM MUST BE A POSITIVE NUMBER.
      IF (X-1.2) 11,11,12
11    XSQ=X*X
      PROB= 0.79788455*X*(0.99999774-XSQ*(0.16659433-XSQ*(0.02463831 0-XS
     1Q*0.00239748 67)))
      RETURN
12    IF(X-2.9) 13,14,14
13    XSQ=X*X
      PROB=1.0
      PTERM=1.0
      FACTOR=1.0
```

```
      ODDINT=3.0
970   PTERM=-PTERM*XSQ/(2.0*FACTOR)
      TERM=PTERM/ODDINT
      PROB=PROB+TERM
      IF( ABS (TERM) - 0.00007 ) 80,90,90
90    FACTOR =FACTOR+1.0
      ODDINT=ODDINT+2.0
      GO TO 970
80    PROB=0.79788455*X*PROB
      RETURN
14    RECXSQ= 1.0 / (X*X)
      PROB= 1.0 - 0.79788453*EXP(-X*X/2.0)/X* (1.0-RECXSQ*(1. -RECXSQ*(3.
     1 - RECXSQ*(15. - RECXSQ*105. ))))
      RETURN
      END
      SUBROUTINE CHISQA
C-----SUBROUTINE TO FIT A HISTOGRAM TO THE DATA AND PERFORM THE CHI-SQUA
C-----TEST FOR THE NORMAL OR LOG-NORMAL DISTRIBUTIONS.
      COMMON CUMFRQ(100),NDATA,X(100),DEV,XMEAN,CLB(9),CUB(9),FREQ(9),K,
     1 CHISQR,TITLE(2),SUBTITL(2),CSV(9),CEV(9),PCAREA(100),DSTAT(100),
     2  AREA(9),REQAREA(9),EXFREQ(9),U(9),Z(100),NX(100),DATA,AKURCY,
     3 XMAX,XMIN,PSI,CD,D,R,XLENGTH,YLENGTH,YMAX,YMIN,XMA,XMI,H,COM(3),W
      DIMENSION XFREQ(9)
      CHISQR= .0
C-----TO DETERMINE THE NUMBER OF CLASS INTERVALS,K
      K= 1   +3.3  *ALOG10(DATA)
      REALK=K
C-----IN ORDER TO DETERMINE THE RANGE,FIND X(MAX) AND X(MIN)
      XMAX=X(1)
      XMIN= X(1)
      DO 17 I=1,NDATA
      IF( X(I).GT.XMAX ) XMAX = X(I)
17    IF(X(I).LT. XMIN) XMIN=X(I)
       RANGE=  XMAX- XMIN
C------TO DETERMINE THE CLASS INTERVAL WIDTH,W
C-----ROUTINE TO ROUND OFF CLASS WIDTH TO SAME NUMBER OF PLACES AS THE A
      DIVIDE = 1.0/AKURCY
   20 KW=(((RANGE+AKURCY)/REALK)+.5*AKURCY)*DIVIDE
      RK1 = KW
      W = RK1/DIVIDE
      DO 22 I=1,K
      A=I
      B = 0.5*AKURCY
      CSV(I)= XMIN+(A-1.0)*W
      CEV(I)= CSV(I)+W-AKURCY
      CLB(I)= CSV(I)-B
22    CUB( I ) = CEV(I)+B
      CEV(K) = XMAX
      CUB(K) = CEV(K) +B
      DO 23 J=1,K
```

```
      XFREQ(J)=0.0
23    FREQ(J)=0.0
      DO 24 I=1,NDATA
      DO 24 J=1,K
      IF(X(I).GE.CLB(J).AND.X(I).LT.CUB(J)) FREQ(J)=FREQ(J)+1.0
24    CONTINUE
      DO 25 J=1,K
   25 XFREQ(J)=FREQ(J)
      Y=0.
      PRINT 62,XMAX
      PRINT 63,XMIN
      PRINT 65,W
C-----CHI-SQUARE TEST
   26 PRINT 41
      PRINT 406
      DO 30 I=1,K
      Z(I)=( CUB(I)- XMEAN) / DEV
      T= ABS( Z(I) )
30    AREA(I)= PROB(T)/2.0
      REQAREA(1) = 0.5 - AREA(1)
       MANU=K-1
      DO 32 I=2,MANU
      M=I-1
      IF(  (Z(I).GE.0.0.AND.Z(M).GE.0.0).OR.( Z(I).LE.0.0.AND.Z(M).LE.0.
     1 )   ) GO TO 31
      REQAREA(I)= AREA(I)+AREA(M)
      GO TO 32
31    REQAREA(I) = ABS( AREA(I)-AREA(M) )
32    CONTINUE
      REQAREA(K)= 0.5-AREA(K-1)
      DO 80 M=1,K
   80 EXFREQ(M)=DATA*REQAREA(M)
      I=1
 2420 IF(FREQ(I).GE.5.) GO TO 2430
      EXFREQ(I+1)=EXFREQ(I+1)+EXFREQ(I)
      FREQ(I+1)=FREQ(I+1)+FREQ(I)
      J=I
      DO 2425 L=1,J
      EXFREQ(L)=0.
 2425 FREQ(L)=0.
      I=I+1
      GO TO 2420
 2430 I=K
 2440 IF(FREQ(I).GE.5.) GO TO 2450
      EXFREQ(I-1)=EXFREQ(I-1)+EXFREQ(I)
      FREQ(I-1)=FREQ(I-1)+FREQ(I)
      DO 2445 L=I,K
      EXFREQ(L)=0.
 2445 FREQ(L)=0.
      I=I-1
```

```
      GO TO 2440
 2450 CONTINUE
      DO 85 M=1,K
      U(M)=0.0
      IF(EXFREQ(M).EQ.0.) GO TO 85
      U(M)=((EXFREQ(M)-FREQ(M))**2)/EXFREQ(M)
   85 CONTINUE
      DO 90 M=1,K
   90 CHISQR=CHISQR+U(M)
C-----TO PRINT THE TABLE FOR CHI-SQUARE TEST
      DO 33 I=1,K
33    PRINT 34,I,CLB(I),CUB(I), EXFREQ(I),FREQ(I),U(I)
      PRINT 35, CHISQR
      DO 150 I=1,K
  150 FREQ(I)=XFREQ(I)
   62 FORMAT(10X,*MAXIMUM VALUE=*,F13.4)
   63 FORMAT(10X*MINIMUM VALUE=*,F13.4)
   65 FORMAT(10X,*CLASS WIDTH=*,F15.4)
41    FORMAT(1H0)
406   FORMAT (8X,5H CELL.10X,10HLOWER CELL,11X,10HUPPER CELL,13X,8HEXPEC
     1TED,13X,8HOBSERVED,13X,11HCHI-SQUARED/8X,6HNUMBER,10X,8HBOUNDARY,
     213X, 8HBOUNDARY,13X,9HFREQUENCY,12X,9HFREQUENCY,12X,13HVALUE OF CE
     3LL/)
   34 FORMAT(10X,I2,F20.4,4F21.4)
   35 FORMAT(*0*,77X,*TOTAL CHI-SQUARED VALUE =*,F13.4)
      RETURN
      END
      SUBROUTINE DTEST
C-----SUBROUTINE TO CALCULATE THE KOLMOGOROV-SMIRNOV D-VALUES.
      COMMON CUMFRQ(100),NDATA,X(100),DEV,XMEAN,CLB(9),CUB(9),FREQ(9),K,
     1 CHISQR,TITLE(2),SUBTITL(2),CSV(9).CEV(9),PCAREA(100),DSTAT(100),
     2  AREA(9),REQAREA(9),EXFREQ(9),U(9),Z(100),NX(100),DATA,AKURCY,
     3 XMAX,XMIN,PSI,CD,D,R,XLENGTH,YLENGTH,YMAX,YMIN,XMA,XMI,
     4 H,COM(3),W,ALPHA3,ALPHA4,FOOT(30),IT,SKS
      INTEGER TITLE,SUBTITL
      DO 706 I=1, NDATA
      Z(I) = (X(I) - XMEAN)/DEV
      IF (Z(I)) 703, 704, 705
703   T =ABS(Z(I))
C-----ARNCN=AREA UNDER THE NORMAL CURVE TO LEFT OF Z FOR NEGATIVE Z.
      ARUNCN = (1.0-PROB(T))/2.0
      DSTAT(I) = ARUNCN - PCAREA(I)
      GO TO 706
704   DSTAT(I) = .5 - PCAREA(I)
      GO TO 706
705   T = Z(I)
C-----ARUNCP=AREA UNDER THE NORMAL CURVE TO LEFT OF Z FOR POSITIVE Z.
      ARUNCP = PROB(T)/2.0 +  .500
      DSTAT(I) =    (ARUNCP - PCAREA(I))
706   CONTINUE
```

```
      DO 90 I=1,NDATA
   90 DSTAT(I)=ABS(DSTAT(I))
      SKS=DSTAT(1)
      DO 100 I=2,NDATA
      IF(DSTAT(I).GT.SKS) SKS=DSTAT(I)
  100 CONTINUE
      PRINT 708,SUBTITL
      PRINT 707, (DSTAT(I),I=1,NDATA)
  707 FORMAT(6F20.5)
708   FORMAT (//40X,53H D VALUES FOR KOLMOGOROV-SMIRNOV GOODNESS OF FIT
     1TEST,/,44X,*LISTED IN THE SAME ORDER AS *,2A10,/)
      RETURN
      END
      SUBROUTINE ALPHA
      COMMON CUMFRQ(100),NDATA,X(100),DEV,XMEAN,CLB(9),CUB(9),FREQ(9),K,
     1 CHISQR,TITLE(2),SUBTITL(2),CSV(9),CEV(9),PCAREA(100),DSTAT(100),
     2  AREA(9),REQAREA(9),EXFREQ(9),U(9),Z(100),NX(100),DATA,AKURCY,
     3  XMAX,XMIN,PSI,CD,D,RATIO,XLENGTH,YLENGTH,YMAX,YMIN,XMA,XMI,
     4 H,COM(3),W,ALPHA3,ALPHA4,FOOT(30),IT,SKS
C-----SUBROUTINE TO CALCULATE THE COEFFICIENTS OF SKEWNESS AND KURTOSIS
C-----CALCULATE THE THIRD MOMENT OF THE DATA (SKEWNESS)
      TOP3 = 0.0
      VAR = 0.0
      DO 710 I =1, NDATA
      VAR = VAR + (X(I) - XMEAN)**2
710   TOP3 = TOP3 +(X(I) - XMEAN)**3
      SKEW = TOP3 / DATA
      STDEV = SQRT(VAR/DATA)
C-----ALPHA3 = MOMENT COEFFICIENT OF SKEWNESS.
      ALPHA3 = SKEW/(STDEV**3)
C-----CALCULATE THE FOURTH MOMENT OF THE DATA (KURTOSIS).
      TOP4 = 0.0
      DO 711 I = 1, NDATA
711   TOP4 = TOP4 + (X(I) - XMEAN)**4
      TKURT = TOP4 / DATA
C-----ALPHA4 = MOMENT COEFFICIENT OF KURTOSIS.
      ALPHA4 = TKURT/(STDEV**4)
      PRINT 712
      PRINT 713
      PRINT 714, ALPHA3, ALPHA4
712   FORMAT (///19X,39HMOMENT COEFFICIENT OF SKEWNESS (ALPHA3),15X,39HM
     1OMENT COEFFICIENT OF KURTOSIS (ALPHA4)/)
  713 FORMAT(21X,*FOR NORMAL DISTRIBUTION ALPHA3 = 0.0*,21X,*FOR NORMAL
     1DISTRIBUTION ALPHA4 = 3.0*,/)
714   FORMAT (28X,25HFOR ABOVE DATA---ALPHA3 =F6.3,26X,25HFOR ABOVE DATA
     1---ALPHA4 =,F6.3)
      RETURN
      END
      SUBROUTINE GRAPH
      COMMON CUMFRQ(100),NDATA,X(100),DEV,XMEAN,CLB(9),CUB(9),FREQ(9),K,
```

```
     1 CHISQR,TITLE(2),SUBTITL(2),CSV(9),CEV(9),PCAREA(100),DSTAT(100),
     2 AREA(9),REQAREA(9),EXFREQ(9),U(9),Z(100),NX(100),DATA,AKURCY,
     3  XMAX,XMIN,PSI,CD,D,R,XLENGTH,YLENGTH,YMAX,YMIN,XMA,XMI,
     4 H,COM(3),W,ALPHA3,ALPHA4,FOOT(30),IT,SKS,UNIT
C     DETERMINE DEFAULTS OR SPECIFIED PARAMETERS
C
      IF(XLENGTH.EQ.0.) XLENGTH=6.
      IF(YLENGTH.EQ.0.) YLENGTH=5.
      IF(YLENGTH.GT.5.) YLENGTH=5.
      DO 1 I=1,K
      IF(YMAX.LT.FREQ(I)) YMAX=FREQ(I)
    1 CONTINUE
      I=YMAX
      IF((I/2*2).NE.I) YMAX=YMAX+1.
    6 XMIN = XMIN - 0.10 * (XMAX - XMIN)
    8 IF(XMIN .LT. 0.0) XMIN = 0.0
      XDIF=XMAX-XMIN
      H = 0.15
C
C     DETERMINE SCALING FACTORS
C
      XSCALE = (XMAX - XMIN) / XLENGTH
      YSCALE = YMAX / YLENGTH
      E = IFIX(ALOG10(XMAX - XMIN))
      STP = 10.0 ** E
C
C     LOCATE PLOTTER PEN
C
      CALL PLOT(XLENGTH+2.,0.,-3)
      CALL PLOT(0.,-11.,-3)
      CALL PLOT (0.,5.,-3)
C
C     CONSTRUCT Y-AXIS
C
      CALL PLOT (0.,YLENGTH,2)
      DIV=1./YSCALE
   21 IF(DIV.GE.(2.*H)) GO TO 22
      DIV=2.*DIV
      GO TO 21
   22 STEP=DIV
      YY=0.
   23 CALL PLOT (.05,YY,3)
      CALL PLOT (-.05,YY,2)
      YN=YY*YSCALE
      CALL NUMBER (-.3,YY,H,YN,0.,-1)
      YY=YY+STEP
      IF(YY.LE.(YLENGTH+.01)) GO TO 23
      YY=(YLENGTH-3.75)/2.
      IF (YY.LT.0.) YY=0.
      CALL SYMBOL (-.4,YY,H,30HFREQUENCY/CLASS INTERVAL WIDTH,90.,30)
```

```
C
C     CONSTRUCT, DRAW, AND LABEL X-AXIS
C
      CALL PLOT (0.,0.,3)
      IF(IT.EQ.1HL) GO TO 25
      XMIN=IFIX(XMIN/STP)
      XMAX=IFIX(XMAX/STP+1.0)
      XSCALE  =  ( XMAX - XMIN ) / XLENGTH * STP
   25 XDIF=XMAX-XMIN
      DIV=10.*XDIF/XLENGTH
      IF(XMIN.EQ.0.) GO TC 26
      CALL PLOT (.3,0.,2)
      CALL PLOT (.35,0.,3)
      CALL PLOT (XLENGTH,0.,2)
      CALL PLOT (.35,.05,3)
      CALL PLOT (.25,-.05,2)
      CALL PLOT (.3,-.05,3)
      CALL PLOT (.4,.05,2)
      GO TO 28
   26 CALL PLOT (XLENGTH,0.,2)
   28 IF(DIV.LT.12.7) GO TO 30
      DIV=DIV/10.
      GO TO 28
   30 DIV=DIV/10.
      IF(DIV.LT.0.2) DIV=DIV*10.
   32 CALL NUMBER (0.,-.2,H,0.,0.,0)
      XX=0.
      DO 35 I=1,25
      XX=XX+1./DIV
      IF(XX.GT.XLENGTH) 40,33
   33 CALL PLOT (XX,.05,3)
      CALL PLOT (XX,-.05,2)
      XN=XMIN+I*XDIF/(DIV*XLENGTH)
      IF(IT.EQ.1HL) GO TO 37
      CALL NUMBER (XX-.1,-.2,H,XN,0.,0)
   35 CONTINUE
   40 DO 41 I=1,2
      IF(SUBTITL(I).EQ.1H ) GO TO 42
   41 CONTINUE
      I=2
      GO TO 43
   37 CALL NUMBER (XX-.2,-.2,H,XN,0.,2)
      GO TO 35
   42 I=I-1
   43 XX=(XLENGTH-I)/2.
      I=I*10
      CALL SYMBOL (XX,-.5,.15,SUBTITL(1),0.,I)
      IF(IT.EQ.1HL) GO TO 46
      IF(STP.LT.1.01) GO TO 48
      CALL WHERE (XX,YY)
      CALL SYMBOL (XX,-.5,H,5H X 10,0.,5)
```

```
      CALL WHERE (XX,YY)
      CALL NUMBER (XX+H,YY+.5*H,.5*H,E,0.,-1)
C
C     CONSTRUCT AND DRAW HISTOGRAM
C
      GO TO 48
   46 DO 47 I=1,K
      CLB(I)=(CLB(I)-XMIN)/XSCALE
   47 CUB(I)=(CUB(I)-XMIN)/XSCALE
      GO TO 52
   48 DO 50 I=1,K
      CLB(I)=((CLB(I)/STP)-XMIN)/(XSCALE/STP)
   50 CUB(I)=((CUB(I)/STP)-XMIN)/(XSCALE/STP)
   52 CALL PLOT (CLB(1),0.,3)
      DO 55 I=1,K
      Y=FREQ(I)/YSCALE
      CALL PLOT (CLB(I),Y,2)
      CALL PLOT (CUB(I),Y,2)
   54 CALL PLOT (CUB(I),0.,2)
   55 CONTINUE
C
C     COMPUTE AND DRAW NORMAL CURVE ONE POINT AT A TIME
C
   60 STEP=XDIF*STP/100.
      IF(IT.EQ.1HL) STEP=STEP/STP
      C=1./(DEV*2.50665)
      XX=XMIN
      IF(IT.EQ.1HN) XX=XMIN*STP
      CALL PLOT(0.,0.,3)
      DO 100 I=1,100
      Y=C*EXP(-.5*(XX-XMEAN)**2/DEV**2)/YSCALE*NDATA*H
      XU=(XX-XMIN*STP)/XSCALE
      IF(IT.EQ.1HL) XU=(XX-XMIN)/XSCALE
      IF(XU.GT.20.) GO TO 100
      IF(XMIN.EQ.0.) 80,70
   70 IF(XU.GE.0.4) 80,90
   80 CALL PLOT (XU,Y,2)
      GO TO 100
   90 CALL PLOT (XU,Y,3)
  100 XX=XX+STEP
C
C     OTHER ALPHA-NUMERIC COMMENTARY
C
  130 CALL PLOT(0.,-1.,-3)
      CALL SYMBOL (0.,0.,H,11HMEAN VALUE:,0.,11)
      CALL SYMBOL(0.,-2.*H,H,19HSTANDARD DEVIATION:,0.,19)
      CALL SYMBOL (0.,-4.*H,H,24HKOLMOGOROV-SMIRNOV TEST:,0.,24)
      CALL WHERE(XX,YY)
      XX=XX+H
      CALL SYMBOL (0.,-6.*H,H,17HCHI-SQUARED TEST:,0.,17)
      CALL SYMBOL (0.,-8.*H,H,9HSKEWNESS:,0.,9)
```

```
      CALL SYMBOL(0.,-10.*H,H,9HKURTOSIS:,0.,9)
      IF(IT.EQ.1HN) CALL NUMBER (XX,0.,H,XMEAN,0.,1)
      IF(IT.EQ.1HL) CALL NUMBER(XX,0.,H,XMEAN,0.,3)
      CALL WHERE(XIN,YY)
      XIN=XIN+2.*H
      IF(IT.EQ.1HN) CALL NUMBER(XX,-2.*H,H,DEV,0.,1)
      IF(IT.EQ.1HL) CALL NUMBER(XX,-2.*H,H,DEV,0.,3)
      CALL NUMBER(XX,-4.*H,H,SKS,0.,3)
      CALL NUMBER(XX,-6.*H,H,CHISQR,0.,3)
      CALL NUMBER(XX,-8.*H,H,ALPHA3,0.,3)
      CALL NUMBER(XX,-10.*H,H,ALPHA4,0.,3)
      CALL SYMBOL (XIN,0.,H,UNIT,0.,10)
      CALL SYMBOL (XIN,-2.*H,H,UNIT,0.,10)
      CALL SYMBOL(0.,-14.*H,H,FOOT(1),0.,50)
      CALL SYMBOL(0.,-16.*H,H,FOOT(6),0.,50)
      CALL SYMBOL(0.,-18.*H,H,FOOT(11),0.,50)
      CALL SYMBOL(0.,-20.*H,H,FOOT(16),0.,50)
      CALL SYMBOL(0.,-22.*H,H,FOOT(21),0.,50)
      CALL SYMBOL(0.,-24.*H,H,FOOT(26),0.,50)
      CALL PLOT(XLENGTH+2.,0.,3)
      IF(IT.EQ.1HL) GO TO 150
      XX=(XLENGTH-3.75)/2.
      IF(XX.LT.0.) XX=0.
      CALL SYMBOL (XX,6.25,H,30HNORMAL DISTRIBUTION PARAMETERS,0.,30)
      RETURN
  150 XX=(XLENGTH-4.25)/2.
      IF(XX.LT.0.) XX=0.
      CALL SYMBOL (XX,6.25,H,34HLOG NORMAL DISTRIBUTION PARAMETERS,0.,
     1 34)
      RETURN
      END
```

APPENDIX D

PROGRAM WEIBULL

This program calculates the three Weibull distribution parameters (β, η, and γ) from cycles-to-failure data. It uses those parameters to calculate cycle life for 99% and 90% reliability with a 90% confidence interval. It also performs the Chi-squared and Kolmogorov-Smirnov goodness-of-fit tests.

The program input consists of:

1. A header card - to identify the data block.
2. A set of data cards (in increasing order of cycles to failure)
3. Trailer card - to separate data blocks.

The input format for the header card is:

a.	Columns	2-7	date code in alphameric format
b.		8-9	blank
c.		10-40	run identification in alphameric format
d.		41-46	number of data (sample size) in fixed point format
e.		48-52	minimum life increment in fixed point format
f.		53-80	not used

Example:

Dec 71 WEIBULL SL = 100,000 SR = Infinity 35 100

The input format for the data cards is:

a.	Columns	2-7	cycles to failure in fixed point format
b.		9-15	component life in fixed point format (same as for individual fatigue life tests).

c. 20-23 date code (prints out only) in alphameric format

d. 25-27 number of suspensions in fixed point format (either failures just removed from test, or remaining good items when testing by groups).

e. 28-31 blank.

32-35 test code in alphameric format (Note: 28-42 simply prints as punched and can all be left blank or used as comments space).

36-39 lot code in alphameric format.

40-43 blank.

44-80 not used.

The input format for the trailer card is:

a. Columns 1-8 not used.

9-12 punched: - 1.0 in floating point format.

13-80 not used.

By use of the trailer card, the program is written to process as many sets of data as desired. An end of file card is used to signal the end of the last data deck.

Following is a list of important variables and symbols used in program WEIBULL:

List of Definitions for Program WEIBULL

Main Program:

NDAT	=	number of data points
W	=	NDATA + NODATA = DATA = W = V = J = K = FNZ = NODAT
IACTL	=	cycles of test at which a failure occurs
IX=Z	=	cycles of life of specimen at failure
MINL	=	IACTL
INCR	=	minimum life increment
C	=	cumulative life increment
R	=	median rank
R_1	=	log(log) median rank
Q_1	=	log (X - G)
B_1	=	B = weibull slope, β
BETA	=	inverse of slope
E_1	=	E = goodness of fit of regression line
G_1	=	minimum life parameter, γ
U	=	median life
ETA	=	scale parameter = η
CG	=	plus confidence interval
ONE	=	1% life = 99% reliability
CONE	=	upper confidence limit on 1% life
CONM	=	lower confidence limit on 1% life
TEN	=	10% life = 90% reliability
CTEN	=	upper confidence limit on 10% life
CTENM	=	lower confidence limit on 10% life
CU	=	upper confidence limit on median life (U)
CUM	=	lower confidence limit on median life

Subroutine for Kolmogorov-Smirnov Test

(Subroutine DTEST)

Z(I)	=	Weibull cumulative frequency distribution
CUMFREQ(I)	=	cumulative observations (number of failures)
PERCF(I)	=	data cumulative frequency as percentage of failures
DSTAT(I)	=	absolute difference between the data cumulative frequency and the hypothesized cumulative frequency

Subroutine for Chi-squared test (Subroutines CHISQA and CHISQB)

K	=	number of cells
XMAX	=	largest value of cycles to failure
XMIN	=	smallest value of cycles to failure
CSV	=	cell starting value
CEV	=	cell end value
CLB	=	cell lower bound
CUB	=	cell upper bound
FREQ(J)	=	number of observations in J^{th} cell
REQAREA(J)	=	expected value of J^{th} cell (percentage)
EXFREQ(J)	=	expected number of observations

The program WEIBULL listing in Fortran language follows:

```
C-----WEIBULL
C-----WEIBULL
C-----WEIBULL
      PROGRAM WEIBULL (INPUT,OUTPUT,TAPE2=INPUT,TAPE3=OUTPUT,
     1  TAPE1,TAPE4,TAPE5,PLOT=TAPE1)
      DIMENSION Y(100),FREQ(9),CLB(9),CUB(9),DSTAT(100),CUMFRQ(100),
     1 PERCF(100)
      COMMON/N4/ X, R
      COMMON /N5/ IACTL, IX, A1, A2, NOR, A3, A4, A5, A6
  1   J = 0
      K = 0
      I = 0
      ICOUN = 0
      NOD = 0
      XPREV = 0.0
      RPREV = 0.0
      REWIND 4
      REWIND 5
      READ (2,10) NODAT, INCR,M
      IF(EOF(2)) 83,80
  10  FORMAT (40H                                        ,2I6,I8)
  80  IF (NODAT-50) 82,83,83
  82  WRITE (3,40)
  40  FORMAT (5H1DATE,15X,9HRUN IDENT  )
      WRITE (3,10)
  7   READ(2,20) IACTL, IX, A1, A2, NOR, A3, A4, A5, A6
   20 FORMAT (I7, 1X, I7, 1X, 2A4, I3, 4A4, F16.6)
      IF (IX) 4, 3, 2
  2   J = J + 1
  3   K = K + 1
      I = I + 1
      Y(I) = IACTL
      IF (NOR) 6, 6, 5
  6   NOR = 1
  5   NOD = NOD + NOR
      WRITE (4)   IACTL, IX, A1, A2, NOR, A3, A4, A5, A6
      GO TO 7
  4   IF (NODAT - NOD) 8, 9, 8
C-----
C-----   ERROR STOP - NO. OF DATA NOT CORRECT
C-----
  8   WRITE (3,30) NODAT, NOD
  30  FORMAT (22H1NO. OF DATA INCORRECT   /1H0, I6, 5X, I6 / 1H1)
      REWIND 4
      GO TO 83
  9   REWIND 4
      WRITE (3,220) NODAT, J, INCR,M
 220  FORMAT (1H0,4X,11HNO. OF DATA   I6,10X,13H  NO. OF FAIL   I6,10X,
     115H  MIN LIFE INCR  I6,5X,5H  M =  ,I6//)
      WRITE (3,230)
```

```
230  FORMAT (2X,6HACTUAL,3X,4HCOMP,56X,4HCOMP/
    13X,4HLIFE ,4X,4HLIFE ,9H P  DATE  ,4H NOR,6X,10HTEST  LOT
    211X,11HMEDIAN RANK,8X,4HLIFE //)
     Z = 0.0
     W = NODAT
     NDATA = NODAT
     NODATA = NODAT
     DATA = NODAT
     AKURCY = 1.
     V = W
     READ (4)    IACTL, IX, A1, A2, NOR, A3, A4, A5, A6
     REWIND 4
     MINL = IACTL
     FNZ = J
     IF (MINL - 1) 11, 11, 12
 11  MINL = 1
     INCR = 1
     GO TO 13
 12  MINL = MINL - 1
 13  R1 = 1.0 - 2.**(-1.0/W) + (1.0 - 2.**(1.0 - 1.0/W))/(W - 1.0)
     REWIND 4
     DO 14 I = 1, K
     READ (4)    IACTL, IX, A1, A2, NOR, A3, A4, A5, A6
     X = IX
     IF(X) 16, 16, 17
 16  IF(I - 1) 18, 18, 19
 19  IF(XPREV) 18, 18, 21
 21  R1 = RPREV
 18  K1 = NOR
     DO 22 K2= 1, K1
     V = V - 1.0
     RINV=1.0+(W-1.0)*(R1-1.0+2.0**(-1.0/W))/(2.0**(1.0-1.0/W)-1.0)
 22  Z = RINV + (W + 1.0 - RINV)/(1.0 + V)
     R = 0.
     GO TO 15
 17  IF(I - 1) 24, 24, 23
 23  IF(XPREV) 26, 26, 24
 24  Z = Z + (W + 1.0 - Z)/(1.0 + V)
 26  R=1.0-2.0**(-1.0/W)+((Z-1.0)/ (W-1.0))*(2.0**(1.0-1.0/W)-1.0)
     V = V - 1.0
 15  XPREV = X
     RPREV = R
     WRITE(3,241) IACTL, IX, A1, A2, NOR, A3, A4, A5, A6,R,X
 241 FORMAT(1X,I7,1X,I7,1X,2A4,1X,I3,3X,4A4,F16.6,F12.0)
     ICOUN = ICOUN + 1
     IF (ICOUN  - 68) 14, 41, 41
 41  ICOUN = 0
     WRITE(3,400)
 400 FORMAT (1H1)
 14  WRITE (5) X, R
     REWIND 4
```

```
      REWIND 5
      E1 = 0.0
      IF(Y(1) .LE. 1000) GO TO 117
      IF(Y(1) .GT. 1000 .AND. Y(1) .LE. 10000) GO TO 116
      IF(Y(1) .GT. 10000) INCR = 10000
      GO TO 127
  116 INCR = 1000
      GO TO 127
  117 INCR = 100
      IF(M .GT. 1000) M = M - 1000
  127 E1 = 0.0
      DO 27 KN=M,MINL,INCR
      QSUM = 0.0
      RSUM = 0.0
      QSOS = 0.0
      RSOS = 0.0
      PROD = 0.0
      G = KN - 1
      DO 28 I = 1, K
      READ (5) X,R
      IF(X) 28, 28, 29
   29 IF(X - G) 31, 31, 32
C-----
C-----   NEGATIVE ARGUMENT IN LOG. FUNCTION
C-----
   31 WRITE (3,100)
  100 FORMAT(1H1,26HNEG. ARG. IN LOG. FUNCTION /,1H1)
      GO TO 83
   32 Q1 = ALOG(X - G)
      IF(R) 31, 31, 34
C-----
C-----   NEGATIVE ARGUMENT IN LOG FUNCTION
C-----
   34 R1= ALOG(ALOG(1.0/(1.0 - R)))
      QSUM = QSUM + Q1
      RSUM = RSUM + R1
      QSOS = QSOS + Q1 * Q1
      RSOS = RSOS + R1 * R1
      PROD = PROD + Q1 * R1
   28 CONTINUE
      REWIND 5
      B = (FNZ * PROD - QSUM * RSUM)/(FNZ * QSOS - QSUM * QSUM)
      A = (RSUM - B * QSUM) / FNZ
      C = SQRT((FNZ * QSOS - QSUM * QSUM) * (FNZ * RSOS - RSUM * RSUM)
      E = (FNZ*PROD - QSUM * RSUM)/C
      IF(E - E1) 37, 36, 36
   36 E1 = E
      G1 = G
      B1 = B
      A1 = A
```

```
 27  CONTINUE
  37 M = G + 1 - INCR
  39 IF(INCR - 1000) 38,117,116
  38 ARG = EXP ( -A1)
     BETA = 1.0/B1
     ETA = (0.99967*ARG)**BETA
     U = G1 + (0.69315*ARG)**BETA
     ONE = G1 + (0.01005*ARG)**BETA
     TEN = G1 + (0.105*ARG)**BETA
     B = B1 * (1.0 + 1.163 / SQRT(W))
     CG=G1+.5*(U-G1)*(4.32159**(1.0/B)-0.074**(1.0/B))/(W**(1.0/B))
     CONST = (1.645 * U) / (SQRT(W) * 0.69315 ** BETA)
     FACT = CONST * 10.010038 * (0.995E-02**BETA/B1)
     CONE = ONE + FACT
     CONM = ONE - FACT
     FACT = CONST * 3.1924748 * (0.104360**BETA/B1)
     CTEN = TEN + FACT
     CTENM = TEN - FACT
     FACT = CONST * 0.69315**BETA/(B1 * 0.69315)
     CU = U + FACT
     CUM = U - FACT
     WRITE (3,450) B1, E1, ETA, G1,CG, ONE, CONE, CONM
     WRITE (3,451) TEN, CTEN, CTENM, U, CU, CUM
450  FORMAT (// 12X,13HWEIBULL SLOPE,5X,15HGOODNESS OF FIT ,5X,
    115HSCALE PARAMETER,/5X,3F20.5,//
    213X,12HMINIMUM LIFE ,11X,9HPLUS CONF,/
    35X,2F20.5,//
    49X,16HONE PERCENT LIFE ,11X,9HPLUS CONF ,10X,10HMINUS CONF ,/
    55X,3F20.5 //)
451  FORMAT (1H+,9X,16HTEN PERCENT LIFE,11X,9HPLUS CONF,10X,
    111H MINUS CONF ,/5X,3F20.5,//
    214X,11HMEDIAN LIFE ,11X,9HPLUS CONF ,10X, 10HMINUS CONF ,/
    35X,3F20.5 //)
     CALL DTEST (Y,B1,G1,U,NODATA,DSTAT,PERCF,CUMFRQ,ETA,SKSTAT)
     CALL CHISQA (Y,DATA,NDATA,PROB,AKURCY,XMEAN,DEV,Z,G1,B1,ETA,FREQ,
    1 XMAX,XMIN,CLB,CUB,NUMINTS,CELLWD)
     CALL CHISQB (Y,NDATA,G1,B1,ETA,CHISQR)
     CALL GRAPH (FREQ,XMAX,XMIN,CLB,CUB,SKSTAT,CHISQR,B1,ETA,G1,NDATA,
    1 NUMINTS,CELLWD)
     GO TO 1
  83 CALL PLOT (0.0, 0.0, 999)
     CALL EXIT
     END
```

```
      SUBROUTINE DTEST (Y,B1,G1,U,NODATA,DSTAT,PERCF,CUMFRQ,ETA,SKSTAT)
      DIMENSION Z(100),Y(100),DSTAT(100),PERCF(100),CUMFRQ(100)
C
C     SUBROUTINE TO CALCULATE THE KOLMOGOROV-SMIRNOW D-VALUES
C
      DO 500 I=1,NODATA
      Z(I) = 1.0 - EXP (-(((Y(I)-G1)/ETA)**B1))
  500 CONTINUE
C
C     SET CUMFRQ(2) ARRAY
C
      DO 501 I=1,NODATA
  501 CUMFRQ(I) = I
C
C     PERCF = F(N) OF THE NUMBER OF DATA
C
      DO 502 I=1,NODATA
  502 PERCF(I) = CUMFRQ(I)/NODATA
      DO 503 I=1,NODATA
  503 DSTAT(I) = Z(I) - PERCF(I)
      PRINT 520
      PRINT 521, (DSTAT(I),I=1,NODATA)
  521 FORMAT (6(10X,F10.5))
  520 FORMAT (//40X,53H D VALUES FOR KOLMOGOROV-SMIRNOV GOODNESS OF FIT
     1TEST/41X,52H(LISTED IN THE SAME ORDER AS CYCLES-TO-FAILURE DATA)/)
      SKSTAT = 0.0
      DO 10 I=1,NODATA
      IF(ABS(DSTAT(I)).GT.SKSTAT) SKSTAT = ABS(DSTAT(I))
   10 CONTINUE
      PRINT 400, SKSTAT
  400 FORMAT(//,10X,*KOLMOGOROV-SMIRNOV TEST RESULT = *,F8.5,/)
      RETURN
      END
```

```
      SUBROUTINE CHISQA(X,DATA,NDATA,PROB,AKURCY,XMEAN,DEV,Z
     1,G1,B1,ETA,FREQ,XMAX,XMIN,CLB,CUB,K,W)
C-----SUBROUTINE TO FIT A HISTOGRAM TO THE DATA AND PERFORM THE CHI-SQUA
C-----TEST FOR THE WEIBULL DISTRIBUTION
      DIMENSION X(50), CSV(9), CEV(9), CLB(9), CUB(9),
     1REQAREA(9),AREA(9),EXFREQ(9),     FREQ(9),     U(9)
      CHISQR= .0
C-----TO DETERMINE THE NUMBER OF CLASS INTERVALS,K
      K = 1.0 + 3.3*ALOG10(DATA)
      REALK=K
C-----IN ORDER TO DETERMINE THE RANGE,FIND X(MAX) AND X(MIN)
      XMAX=X(1)
      XMIN= X(1)
      DO 17 I=1,NDATA
      IF( X(I).GT.XMAX ) XMAX = X(I)
17    IF(X(I).LT. XMIN) XMIN=X(I)
       RANGE=  XMAX- XMIN
C------TO DETERMINE THE CLASS INTERVAL WIDTH,W
C-----ROUTINE TO ROUND OFF CLASS WIDTH TO SAME NUMBER OF PLACES AS THE A
      DIVIDE = 1.0/AKURCY
      KW =(((RANGE+AKURCY)/REALK)+.5*AKURCY)*DIVIDE
      RK1 = KW
      W = RK1/DIVIDE
      PRINT 141
      PRINT 241
      PRINT 177, NDATA, G1, B1, ETA
      PRINT 41
      PRINT 62,XMAX
      PRINT 63,XMIN
      PRINT 65,W
      B = 0.5*AKURCY
      DO 22 I=1,K
      A=I
      CSV(I)= XMIN+(A-1.0)*W
      CEV(I)= CSV(I)+W-AKURCY
      CLB(I)= CSV(I)-B
      CUB(I) = CEV(I) +B
   22 CONTINUE
      CEV(K) = XMAX
      CUB(K) = CEV(K) +B
      DO 23 J=1,K
23    FREQ(J)=0.0
      DO 24 I=1,NDATA
      DO 24 J=1,K
      IF(X(I).GE.CLB(J).AND.X(I).LE.CUB(J)) FREQ(J) = FREQ(J) + 1.0
24    CONTINUE
C-----CHI-SQUARE TEST
      PRINT 406
      DO 30 I=1,K
      AREA(I) = 1.0 -EXP(-(((CUB(I)-G1)/ETA)**B1))
```

```
      IF (I .EQ. 1) GO TO 51
      IF (I .GT. 1 .AND. I .LT. K) GO TO 52
      REQAREA(K) = 1.0 - AREA(K)
      GO TO 30
 51   REQAREA(I) = AREA(I)
      GO TO 30
 52   REQAREA(I) = AREA(I) - AREA(I-1)
 30   CONTINUE
 76   DO 80 M = 1,K
      EXFREQ(M)=DATA*REQAREA(M)
      U(M)=(( EXFREQ(M)-FREQ(M))**2)/EXFREQ(M)
80    CHISQR=CHISQR+U(M)
C-----TO PRINT THE TABLE FOR CHI-SQUARE TEST
 88   DO 33 I = 1,K
33    PRINT 34,I,CLB(I),CUB(I), EXFREQ(I),FREQ(I),U(I)
      PRINT 35, CHISQR
62    FORMAT( 10X, 14HMAXIMUM VALUE=,F15.6)
63    FORMAT( 10X, 14HMINIMUM VALUE=, F15.6)
65    FORMAT( 10X, 12HCLASS WIDTH=, F17.6)
41    FORMAT(1H0)
406   FORMAT (8X,5H CELL,10X,10HLOWER CELL,11X,10HUPPER CELL,13X,8HEXPEC
     1TED,13X,8HOBSERVED,13X,11HCHI-SQUARED/8X,6HNUMBER,10X,8HBOUNDARY,
     213X, 8HBOUNDARY,13X,9HFREQUENCY,12X,9HFREQUENCY,12X,13HVALUE OF CE
     3LL/)
34    FORMAT (10X,I2,5F21.6)
35    FORMAT (1H0,81X,25HTOTAL CHI-SQUARED VALUE =,F10.6//)
 141  FORMAT (1H0,70X,*CHI-SQUARED TEST*,//)
 241  FORMAT(1H0,70X,*FIXED CELL WIDTHS*,//)
 177  FORMAT (1H0, 10X, *INPUTS = *,I10, 3F15.3,/)
 78   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CHISQB (X,NDATA,G1,B1,ETA,CHISQR)
      DIMENSION X(50),    CLB(9), CUB(9), REQAREA(9), AREA(9), FREQ(9),
     2EXFREQ(9), U(9)
C-----SUBROUTINE TO FIT A HISTOGRAM TO THE DATA AND PERFORM THE
C-----CHI-SQUARED TEST FOR THE WEIBULL DISTRIBUTION
      PRINT 341
      CHISQR = 0.0
      J = 0
      DO 26 K=5,NDATA,5
      J = J+1
      IF (K .LT. NDATA) CUB(J)=(X(K)+X(K+1))/2.0
      IF (K .EQ. 5) CLB(J)=X(1)
      IF (K .GT. 5 .AND. K .LT. NDATA) CLB (J)=CUB(J-1)
      L = (NDATA-K)
      IF(L.NE.0) AREA(J)=1.0-EXP(-(((CUB(J)-G1)/ETA)**B1))
      IF(L.EQ.0) AREA(J) =1.0
      FREQ(J)=5.0
      IF(J.EQ.1) REQAREA(J)=AREA(J)
      IF(J.GT.1.AND.L.NE.0) REQAREA(J)=AREA(J)-AREA(J-1)
      IF (L.LT.5) GO TO 27
      GO TO 26
   27 IF(L.NE.0) J=J+1
      CUB(J)=X(NDATA)
      CLB(J)=CUB(J-1)
      REQAREA(J)=1.0-AREA(J-1)
      IF( L .NE. 0) FREQ(J) = L
   26 CONTINUE
      I = J
      DO 25 J=1,I
      EXFREQ(J) = NDATA*REQAREA(J)
   25 CONTINUE
      K = I
   62 I = 1
 2420 IF (EXFREQ(I) .GE. 5.) GO TO 2430
      EXFREQ(I+1) = EXFREQ(I+1) + EXFREQ(I)
      FREQ(I+1) = FREQ(I+1) + FREQ(I)
      J = I
      DO 2425 L=1,J
      EXFREQ(L) = 0.
 2425 FREQ(L) = 0.
      I = I+1
      GO TO 2420
 2430 I = K
 2440 IF (EXFREQ(I) .GE. 5.) GO TO 2450
      EXFREQ(I-1) = EXFREQ(I-1)+EXFREQ(I)
      FREQ(I-1) = FREQ(I-1) + FREQ(I)
      DO 2445 L=I,K
      EXFREQ(L) = 0.
 2445 FREQ(L) = 0.
```

```
      I = I+1
      GO TO 2440
 2450 CONTINUE
      DO 85 M=1,K
      U(M) = 0.0
      IF (EXFREQ(M) .EQ. 0.) GO TO 85
      U(M) = ((EXFREQ(M) - FREQ(M))**2/EXFREQ(M))
   85 CONTINUE
      CHISQR = 0.0
      DO 90 M=1,K
   90 CHISQR = CHISQR + U(M)
      J = K
   88 DO 33 I = 1,J
33    PRINT 34,I,CLB(I),CUB(I), EXFREQ(I),FREQ(I),U(I)
      PRINT 35, CHISQR
34    FORMAT (10X,I2,5F21.6)
35    FORMAT (1H0,81X,25HTOTAL CHI-SQUARED VALUE =,F10.6)
  341 FORMAT(1H0,65X,*VARIABLE CELL WIDTHS*,//)
   78 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GRAPH (FREQ,XMAX,XMIN,CLB,CUB,SKS,CHISQR,BETA,ETA,
     1 GAMMA,NDATA,K,CELLWD)
      DIMENSION FREQ(9),CLB(9),CUB(9)
      INTEGER FOOT(30),SUBTITL(2)
      LOGICAL NITIAL
      DATA NITIAL / .TRUE. /
C
C     READ PLOT CARD
C
      READ(2,400) IPLOT,SUBTITL,IPEN
  400 FORMAT(4A10)
      IF(EOF(2)) 999,3
    3 IF(IPLOT.NE.7HWEIBULL) GO TO 999
      IF(NITIAL) 4,5
    4 NITIAL = .FALSE.
      PEN = 0.3
      IF(IPEN .EQ. 10HBALL POINT) PEN = 0.0
      CALL INITIAL (0,1,PEN,0)
    5 READ (2,401) FOOT
  401 FORMAT(5A10)
C
C     DETERMINE DISTANCES
C
    2 XLENGTH = 6.0
      YLENGTH = 5.0
      H = 0.15
      YMAX = 0.0
      DO 1 I=1,K
      IF(YMAX.LT.FREQ(I)) YMAX=FREQ(I)
    1 CONTINUE
      FRINGE = 0.40*NDATA
      FRINL = 0.25*NDATA
      IF(YMAX .GE. FRINL .AND. YMAX .LT. FRINGE) GO TO 7
      IF(YMAX .GE. FRINGE) YMAX = IFIX(YMAX + FRINGE)
      GO TO 6
    7 YMAX = IFIX(YMAX + FRINL)
    6 CONTINUE
      I=YMAX
      IF((I/2*2).NE.I) YMAX=YMAX+1.
      XMIN = XMIN - 0.10 * (XMAX - XMIN)
      XDIF = XMAX - XMIN
C
C     DETERMINE SCALING FACTORS
C
      XSCALE = (XMAX - XMIN) / XLENGTH
      YSCALE = YMAX / YLENGTH
      E = IFIX(ALOG10(XMAX - XMIN))
      STP = 10.0 ** E
```

```
C
C     LOCATE PLOTTER PEN
C
      CALL PLOT(XLENGTH+2.,0.,-3)
      CALL PLOT(0.,-11.,-3)
      CALL PLOT (0.,5.,-3)
```

```
C
C     CONSTRUCT Y-AXIS
C
      CALL PLOT (0.,YLENGTH,2)
      DIV=1./YSCALE
   21 IF(DIV.GE.(2.*H)) GO TO 22
      DIV=2.*DIV
      GO TO 21
   22 STEP=DIV
      YY=0.
   23 CALL PLOT (.05,YY,3)
      CALL PLOT (-.05,YY,2)
      YN=YY*YSCALE
      CALL NUMBER (-.3,YY,H,YN,0.,-1)
      YY=YY+STEP
      IF(YY.LE.(YLENGTH+.01)) GO TO 23
      YY=(YLENGTH-3.75)/2.
      IF (YY.LT.0.) YY=0.
      CALL SYMBOL (-.4,YY,H,30HFREQUENCY/CLASS INTERVAL WIDTH,90.,30)
C
C     CONSTRUCT, DRAW, AND LABEL X-AXIS
C
      CALL PLOT (0.,0.,3)
      XMIN=IFIX(XMIN/STP)
      XMAX = IFIX(XMAX / STP + 1.0)
      XDIF = XMAX - XMIN
      XSCALE = XDIF / XLENGTH * STP
      DIV=10.*XDIF/XLENGTH
      IF(XMIN.EQ.0.) GO TO 26
      CALL PLOT (.3,0.,2)
      CALL PLOT (.35,0.,3)
      CALL PLOT (XLENGTH,0.,2)
      CALL PLOT (.35,.05,3)
      CALL PLOT (.25,-.05,2)
      CALL PLOT (.3,-.05,3)
      CALL PLOT (.4,.05,2)
      GO TO 28
   26 CALL PLOT (XLENGTH,0.,2)
   28 IF(DIV.LT.12.7) GO TO 30
      DIV=DIV/10.
      GO TO 28
   30 DIV=DIV/10.
      IF(DIV.LT.0.2) DIV=DIV*10.
   32 CALL NUMBER (0.,-.2,H,0.,0.,0)
      XX=0.
      DO 35 I=1,25
      XX=XX+1./DIV
      IF(XX.GT.XLENGTH) 40,33
```

```
33 CALL PLOT (XX,.05,3)
   CALL PLOT (XX,-.05,2)
   XN=XMIN+I*XDIF/(DIV*XLENGTH)
   CALL NUMBER (XX-.1,-.2,H,XN,0.,0)
35 CONTINUE
40 DO 41 I=1,2
   IF(SUBTITL(I).EQ.1H ) GO TO 42
   41 CONTINUE
      I=2
      GO TO 43
   42 I=I-1
   43 XX=(XLENGTH-I)/2.
      I=I*10
      CALL SYMBOL (XX,-.5,.15,SUBTITL(1),0.,I)
      IF(SIP.LT.1.01) GO TO 48
      CALL WHERE (XX, YY, IFAKE)
      CALL SYMBOL (XX,-.5,H,5H X 10,0.,5)
      CALL WHERE (XX, YY, IFAKE)
      CALL NUMBER (XX+H,YY+.5*H,.5*H,E.0.,-1)
C
C     CONSTRUCT AND DRAW HISTOGRAM
C
   48 DO 50 I=1,K
      CLB(I)=((CLB(I)/STP)-XMIN)/(XSCALE/STP)
   50 CUB(I)=((CUB(I)/STP)-XMIN)/(XSCALE/STP)
   52 CALL PLOT (CLB(1),0.,3)
      DO 55 I=1,K
      Y=FREQ(I)/YSCALE
      CALL PLOT (CLB(I),Y,2)
      CALL PLOT (CUB(I),Y,2)
   54 CALL PLOT (CUB(I),0.,2)
   55 CONTINUE
C
C     COMPUTE AND DRAW NORMAL CURVE ONE POINT AT A TIME
C
   60 STEP = XDIF * STP / 150.0
      XX = XMIN * STP
      CALL PLOT(0.,0.,3)
      EACT = EXP(-BETA)
      CHK = EXP(-1.0)
      IF(BETA .LT. 1.0) YSCALE=YSCALE*EACT/CHK
      BE = BETA / ETA
      IDOIT = 2HNO
      DO 100 I = 1,150
      IF(GAMMA .GT. XX) GO TO 100
      Z = (XX - GAMMA) / ETA
      Y = BE * Z ** (BETA - 1.0) * EXP( - 1.0 * Z ** BETA)
```

```
      Y = Y / YSCALE * NDATA * CELLWD
      XU=(XX-XMIN*STP)/XSCALE
      IF(IDOIT .EQ. 2HNO) GO TO 90
      IF(XU.GT.20.) GO TO 100
      IF(XMIN.EQ.0.) 80,70
   70 IF(XU.GE.0.4) 80,90
   80 CALL PLOT (XU,Y,2)
      GO TO 100
   90 CALL PLOT (XU,Y,3)
      IDOIT = 3HYES
  100 XX=XX+STEP
C
C     OTHER ALPHA-NUMERIC COMMENTARY
C
  130 CALL PLOT(0.,-1.,-3)
      CALL SYMBOL (0.0,0.0,H,24HKOLMOGOROV-SMIRNOV TEST:,0.0,24)
      CALL WHERE (XX, YY, IFAKE)
      XX=XX+H
      CALL SYMBOL (0.0,-2.0*H,H,17HCHI-SQUARED TEST:,0.0,17)
      CALL SYMBOL (0.0,-4.0*H,H,21HWEIBULL SLOPE (BETA):,0.0,21)
      CALL SYMBOL (0.0,-6.0*H,H,21HMINIMUM LIFE (GAMMA):,0.0,21)
      CALL SYMBOL (0.0,-8.0*H,H,22HSCALE PARAMETER (ETA):,0.0,22)
      CALL NUMBER (XX,0.0,H,SKS,0.0,3)
      CALL NUMBER (XX,-2.0*H,H,CHISQR,0.0,3)
      CALL NUMBER (XX,-4.0*H,H,BETA,0.0,3)
      CALL NUMBER (XX,-6.0*H,H,GAMMA,0.0,-1)
      CALL NUMBER (XX,-8.0*H,H,ETA,0.0,-1)
      CALL SYMBOL(0.,-14.*H,H,FOOT(1),0.,50)
      CALL SYMBOL(0.,-16.*H,H,FOOT(6),0.,50)
      CALL SYMBOL(0.,-18.*H,H,FOOT(11),0.,50)
      CALL SYMBOL(0.,-20.*H,H,FOOT(16),0.,50)
      CALL SYMBOL(0.,-22.*H,H,FOOT(21),0.,50)
      CALL SYMBOL(0.,-24.*H,H,FOOT(26),0.,50)
      XX=(XLENGTH-3.75)/2.
      IF(XX.LT.0.) XX=0.
      CALL SYMBOL (XX,6.25,H,31HWEIBULL DISTRIBUTION PARAMETERS,0.0,31)
  999 RETURN
      END
```

APPENDIX E

PDP PROGRAM TO CALCULATE ENDURANCE STRENGTH PARAMETERS FROM STAIRCASE TESTS

C-FOCAL, 1969

```
01.10 A "MINIMUM STRESS LEVEL", YP, !
01.20 A "STRESS INCREMENT", DP, !
01.30 A "NO OF SPECIMENS",  NS, !
01.40 T "IF TEST IS BASED ON FAILURES THE CODE IS 0", !
01.50 T "IF BASED ON SUCCESSES THE CODE IS 1", !
01.60 A "WHAT IS THE CODE?", Co, !
01.70 A "NO OF STRESS LEVELS IN TEST", I, !, !
01.74 S CU = 0
01.75 S A = 0
01.76 S B = 0
01.77 T "NO OF SPEC IN EACH LEVL STARTING FROM THE 2ND LOWEST", !
01.80 FOR J = 1, 1, I-1; DO 4.0

02.10 S SD = 1.62*DP*((NS*B-A+2)/NS 2+0.029)
02.20 IF (CO) 2.3,2.3,2.4
02.30 S MU = YP+DP*(A/NS-.5)
02.35 GOTO 2.7
02.40 S MU = YP+DP*(A/NS-.5)
02.70 T %10.03 "MEAN", MU, !, "STD DEV", SD, !
02.80 Q

04.10 A NI, !
04.20 S CU = CU+1
04.30 S A = A+CU*NI
04.40 S B = B+CU 2*NI
```

APPENDIX F

PDP PROGRAM TO CALCULATE PAN WEIGHTS FOR DESIRED BENDING STRESS LEVELS FOR THE ANN ARBOR RESEARCH MACHINE

C-FOCAL, 1969

```
01.10 A "DIAMETER, D, !
01.11 A "LOWER STRESS", A, !
01.12 A "UPPER STRESS", B, !
01.13 A "INCREMENT", I, !
01.15 T "STRESS LEVEL","          MOMENT","              PAN LOAD", !
01.20 F ST=A, I, B,; DO 2.0

02.20 S M=3.1416*D+3*ST/32
02.30 S P=((M+0.267)/4.07)-8.625
02.40 T %8.40,"    ",ST,"    ",M,"    ",P,!

0.350 GOTO 1.10
*
```

REPORT DISTRIBUTION LIST

GRANT NGR 03-002-044

	Copies
National Aeronautics & Space Administration Lewis Research Center 21000 Brookpark Road Cleveland, Ohio 44135	
Attn: J. H. Childs, MS 3-3	1
H. W. Plohr, MS 54-1	1
P. A. Thollot, MS 54-4	1
H. A. Shumaker, MS 54-4	1
N. T. Musial, MS 500-311	1
G. Mandel, MS 6-2	1
Librarian, MS 60-3	2
Report Control Office, MS 5-5	1
V. F. Hlavin, MS 3-14	1
Technology Utilization Office, MS 3-19	1
W. F. Dankhoff, MS 500-111	1
V. R. Lalli, MS 54-4	3
F. H. Stickney, MS 500-203	1
C. R. Ensign, MS 49-1	1
J. P. Gyekenyesi, MS 21-4	1
National Aeronautics and Space Administration Scientific and Technical Information Facility College Park, Maryland 20740	3 plus reproducible
US Air Force Aeropropulsion Laboratory Wright Patterson AFB Dayton, Ohio 45433	
Attn: Library	1
U.S. Army R & D Laboratory Ft. Belvoir, Virginia 22060	
Attn: Library	1
Office of Naval Research Arlington, Virginia 22217	
Attn: Mr. M. K. Ellingsworth C473	1
Library	1
National Science Foundation 1800 G. Street, NW Washington, D.C. 20550	
Attn: Dr. Michael Gaus, RM 340	1
Library	1

REPORT DISTRIBUTION LIST

GRANT NGR 03-002-044

	Copies
U. S. Atomic Energy Commission Technical Reports Library Washington, D. C. 20545	1
NASA Langley Research Center Langley Station Hampton, Va. 23365 Attn: Library	1
NASA Manned Spacecraft Center Houston, Texas 77001 Attn: Library	1
NASA Manned Space Flight Center Huntsville, Ala. 35812 Attn: Library	1
U.S. Atomic Energy Commission Technical Information Service Extension P. O. Box 62 Oak Ridge, Tenn. 37830	3
National Aeronautics and Space Administration NASA Headquarters Program Office Washington, D. C. 20546 Attn: KR/D. E. Negola	1
RNP/F. Scholman	1
Y/F. B. Smith	1
NASA Ames Research Center Ames Research Center Moffett Field, California 94035 Attn: Library	1
NASA Flight Research Center Flight Research Center P. O. Box 273 Edwards, California 93523 Attn: Library	1

REPORT DISTRIBUTION LIST

GRANT NGR 03-002-044

	Copies
NASA Goddard Space Flight Center Goddard Space Flight Center Greenbelt, Md. 20771 Attn: Library	1
Jet Propulsion Laboratory 4800 Oak Groove Drive Pasadena, California 91103 Attn: Library	1
Director Research and Development U.S. Army Weapons Command Rock Island Arsenal, Illinois 61201 Attn: Dr. E. Haug	1
Dr. M. S. Ojalvo Division of Engineering National Science Foundation 1800 G. Street, NW Washington, D. C. 20550	1
Dr. Walter J. Fahey, Dean College of Engineering University of Arizona Tucson, Arizona 85721	1
Dr. George W. Howard, Director Engineering Experiment Station College of Engineering University of Arizona Tucson, Arizona 85721	1
Dr. Lawrence B. Scott, Head Aerospace and Mechanical Engineering Department University of Arizona Tucson, Arizona 85721	1
Louie B. Chester Aerospace and Mechanical Engineering Department University of Arizona Tucson, Arizona 85721	2

REPORT DISTRIBUTION LIST

GRANT NGR 03-002-044

	Copies
Dr. Dimitri B. Kececioglu Aerospace and Mechanical Engineering Department University of Arizona Tucson, Arizona 85721	1